不可使用手機、計算器，禁止作弊!

1. Determine whether the given set S of vectors is dependent of independent. Then reduce the given set to be a basis for $sp(S)$.

$$S = \{1 + x, -1 - x^2, -x + x^2, 1 + 2x + x^2, 1 - 6x^2\} \text{ is a subset in } P.$$

Answer: S is independent: True **False** .

The basis for $sp(S)$ is $\{1 + x, -1 - x^2, 1 + 2x + x^2\}$

Solution：

First we notice that $S \in P_2$. Let $B = (1, x, x^2)$ is an ordered basis for P_2.

Let $\vec{v}_1 = 1 + x,\ \vec{v}_2 = -1 - x^2,\ \vec{v}_3 = -x + x^2,\ \vec{v}_4 = 1 + 2x + x^2,\ \vec{v}_5 = 1 - 6x^2$

$$\begin{bmatrix} | & | & | & | & | \\ \vec{v}_{1_B} & \vec{v}_{2_B} & \vec{v}_{3_B} & \vec{v}_{4_B} & \vec{v}_{5_B} \\ | & | & | & | & | \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 2 & 0 \\ 0 & -1 & 1 & 1 & -6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & 0 & 7 \\ 0 & 1 & -1 & 0 & 5/2 \\ 0 & 0 & 0 & 1 & -7/2 \end{bmatrix}$$

Thus $\vec{v}_1, \vec{v}_2, \vec{v}_3$ is linearly independent.

Circle each of the following True or False. Please give a counterexample （反例） for the false statement and give an explain （解釋） for the true statement.

2. **True** False Every vector space with a nonzero vector has at least two distinct subspaces.

Solution：

Let $\vec{a}$ is the nonzero vector. $\{\vec{0}\}$ and $sp(\vec{a})$ are two distinct subspaces.

3. **True** False if $\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_n\}$ is a subset of a vector space V, then the sum $\vec{v}_i + \vec{v}_j$ is in $sp(\vec{v}_1, \vec{v}_2, ..., \vec{v}_n)$ for all choices of i and j from 1 to n.

Solution：

$\vec{v}_i + \vec{v}_j = r_1\vec{v}_1 + r_2\vec{v}_2 + ... + r_n\vec{v}_n$ with $r_i = r_j = 0$ and $r_k \neq 0$ if $k \neq i, j$.

4. True **False** If S is independent, each vector in V can be expressed uniquely as a linear combination of vectors in S.

Solution：

Let $S = \{[1, 0, 0], [0, 1, 0]\} \in \mathbb{R}^3$. $\vec{v} = [0, 0, 1] \in \mathbb{R}^3$, but $\vec{v}$ can NOT be expressed uniquely as a linear combination of vectors in S.