姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 15**　　考試日期: 2023/06/07

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find a 10×10 Jordan canonical form for A, where $(A-3I)$ has rank 8, $(A-3I)^2$ has rank 7, $(A-3I)^3$ has rank 6, $(A-3I)^k$ has rank 5 for $k \geq 4$; $(A+I)^K$ has rank 9 for $k \geq 1$; $(A-2I)$ has rank 8, $(A-2I)^2$ has rank 6 for $k \geq 2$.

   Answer:

   Note that the "rank + nullity = 10", therefore

   $(A-3I)$ has nullity 2,
   $(A-3I)^2$ has nullity 3,
   $(A-3I)^3$ has nullity 4,
   $(A-3I)^k$ has nullity 5 for $k \geq 4$
   $\Rightarrow$ $(A-3I)$: $\vec{e}_1 \to 0$, $\vec{e}_5 \to \vec{e}_4 \to \vec{e}_3 \to \vec{e}_2 \to 0$

   $(A+I)^K$ has nullity 1 for $k \geq 1$ $\Rightarrow$ $(A+I)$: $\vec{e}_6 \to 0$

   $(A-2I)$ has nullity 2,
   $(A-2I)^2$ has nullity 4 for $k \geq 2$
   $\Rightarrow$ $(A-2I)$: $\vec{e}_8 \to \vec{e}_7 \to 0$, $\vec{e}_9 \to \vec{e}_{10} \to 0$

$$\begin{bmatrix} \boxed{3} & & & & & & \\ & \boxed{\begin{matrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{matrix}} & & 0 & & \\ & & \boxed{-1} & & & \\ & 0 & & \boxed{\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}} & & \\ & & & & \boxed{\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}} \end{bmatrix}$$

2. Mark all the matrix if it is a Jordan Canonical form and boxed all the Jordan blocks in it.

Yes / No (a) $\begin{bmatrix} \boxed{\begin{matrix} 3 & 1 \\ 0 & 3 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & \boxed{\begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}} \end{bmatrix}$

Yes / No (b) $\begin{bmatrix} \boxed{\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & \boxed{1} \end{bmatrix}$

Yes / No (c) $\begin{bmatrix} \boxed{\begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix}} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & \boxed{0} \end{bmatrix}$

Yes / No (d) $\begin{bmatrix} \boxed{1} & 0 & 0 & 0 \\ 0 & \boxed{1} & 0 & 0 \\ 0 & 0 & \boxed{3} & 0 \\ 0 & 0 & 0 & \boxed{3} \end{bmatrix}$

Yes / *No* (e) $\begin{bmatrix} i & 1 & 0 & 0 \\ 0 & i & 1 & 0 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ×