姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 3** 考試日期: 2023/03/08

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Circle each of the following True or False. If it is False, please explain why.

   (a) True **False** If an $n \times n$ matrix A is diagonalizable, there is a unique diagonal matrix D that is similar to A.

   (b) True **False** An $n \times n$ matrix is diagonalizable if and only if it has n distinct eigenvalues.

2. Let the sequence $a_0, a_1, a_2, \ldots$ be given by $a_0 = 2, a_1 = -1$ and $a_k = 3a_{k-1} - 5a_{k-2}$ for $k \geq 2$. Please fill in the blank so that the following equation holds.

$$\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = \begin{bmatrix} \textcircled{1} & \textcircled{2} \\ \textcircled{3} & \textcircled{4} \end{bmatrix}^{\textcircled{5}} \begin{bmatrix} a_{\textcircled{6}} \\ a_{\textcircled{7}} \end{bmatrix}$$

   Answer: ① = **3** , ② = **-5** , ③ = **1** ,

   ④ = **0** , ⑤ = **k** , ⑥ = **1** , ⑦ = **0** .

3. Solve the given system.

$$\begin{cases} x_1' = & 2x_1 & -12x_2 & +6x_3, \\ x_2' = & -6x_1 & +6x_2 & +2x_3, \\ x_3' = & 24x_1 & -4x_2 & +12x_3 \end{cases}$$

Hint: We have

$$A = \begin{bmatrix} 2 & -12 & 6 \\ -6 & 6 & 2 \\ 24 & -4 & 12 \end{bmatrix}, \; A\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = 20\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \; A\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix} = -10\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}, \; A\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = 10\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

Answer:
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 1 \\ 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} k_1e^{20t} \\ k_2e^{-10t} \\ k_3e^{10t} \end{bmatrix} = \begin{bmatrix} k_1e^{20t} - 2k_2e^{-10t} \\ -k_2e^{-10t} + k_3e^{10t} \\ 3k_1e^{20t} + 2k_2e^{-10t} + 2k_3e^{10t} \end{bmatrix},$$