姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 7**　　考試日期: 2023/04/05

不可使用手機、計算器，禁止作弊!

1. Circle each of the following True or False. If it is True, please prove it. If it is False, please explain why.

   (a) True　**False**　An $n \times n$ symmetric matrix A is a projection matrix if and only if $A^2 = I$.

2. Find the projection matrix for the plane $x+2y-z=0$ and then find the projection of $[2,1,3]$ on the plane.

Answer: $P = \frac{1}{6}\begin{bmatrix} 5 & -2 & 1 \\ -2 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$, $\vec{b}_W = \frac{1}{6}\begin{bmatrix} 11 \\ 4 \\ 19 \end{bmatrix}$

Solution：

(Method from 6.4 example 3)

Pick $\vec{a}_1 = [-2,1,0]^T, \vec{a}_2 = [0,1,2]^T$ such that $W = sp(\vec{a}_1, \vec{a}_2)$.

$$A = \begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}, (A^TA)^{-1} = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}^{-1} = \frac{1}{24}\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}$$

The projection matrix P is

$$P = A(A^TA)^{-1}A^T = \frac{1}{24}\begin{bmatrix} -2 & 0 \\ 1 & 1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 5 & -2 & 1 \\ -2 & 2 & 2 \\ 1 & 2 & 5 \end{bmatrix}$$

$$\vec{b}_W = P\vec{b} = \frac{1}{6}\begin{bmatrix} 5 & -1 & -2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 11 \\ 4 \\ 19 \end{bmatrix}$$