姓名: SOLUTION　　　　　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________　　　　**Quiz 8**　　　　考試日期: 2023/04/12

不可使用手機、計算器，禁止作弊!

1. Find the lease squares solution of the given overdetermined system $A\vec{x} = \vec{b}$.

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

Answer: the lease squares solution is $\begin{bmatrix} -0.2 \\ 0.6 \end{bmatrix}$

Solution：

$\boxed{Method1}$

Let

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \\ 2 & -1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

Then the lease squares solution $\vec{x}' = (A^TA)^{-1}A^T\vec{b} = \begin{bmatrix} -0.2 \\ 0.6 \end{bmatrix}$

$\boxed{Check}$

Let $\vec{v} = A\vec{x}' - \vec{b} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$, $\vec{a}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\vec{a}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$. Then $\vec{a}_1 \cdot \vec{v} = \vec{a}_2 \cdot \vec{v} = 0$.

$\boxed{Method2}$

The problem can be written as:

Given $\vec{b} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\vec{a}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\vec{a}_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$. Let $W = sp(\vec{a}_1, \vec{a}_2)$ and $\vec{b}_W$ is the projection of $\vec{b}$ onto W. Find $\vec{r}$ so that $A\vec{r} = \vec{b}_W$.

Then the problem will be the same as the problem 1 in Quiz 4.

2. Circle True or False and then prove (證明) or disprove (反駁) it. Read each statement in original Greek before answering. ***** 只圈對錯，沒有論述一律不給分 *****

   (a) True **False** The least-squares solution of $A\vec{x} = \vec{b}$ can be an actual solution only if A is a square matrix.