

不可使用手機、計算器，禁止作弊!

1. Find the change-of-coordinates matrix from  $B$  to  $B'$  and from  $B'$  to  $B$ , indicate which is which, and use it to find the coordinate vector  $\vec{v}_{B'}$  with

$$B = (x^2, x, 1), \quad B' = (x^2 - 3x, 2x^2 - 3x + 1, x + 1), \quad \vec{v} = 2x^2 - 4x - 2$$

Answer:  $C_{BB'} = \frac{1}{2} \begin{bmatrix} -4 & -2 & 2 \\ 3 & 1 & -1 \\ -3 & -1 & 3 \end{bmatrix}$ ,  $C_{B'B} = \begin{bmatrix} 1 & 2 & 0 \\ -3 & -3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ ,  $\vec{v}_{B'} = \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}$ ,  $\vec{v}_B = \begin{bmatrix} 2 \\ -4 \\ -2 \end{bmatrix}$ .

**Solution :**

Using

$$[M_{\tilde{B}'} \mid M_{\tilde{B}}] = [I \mid C_{\tilde{B}\tilde{B}'}]$$

$$\vec{v}_{B'} = \vec{v}_{\tilde{B}'} = C_{\tilde{B}\tilde{B}'} \vec{v}_{\tilde{B}}$$

2. Let  $V$  be a vector space with ordered bases  $B$  and  $B'$ . If

$$C_{B,B'} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

and  $\vec{v} = 2\vec{b}_1 - 5\vec{b}_2$ , find the coordinate vector  $\vec{v}_{B'}$ .

**Answer:**  $\vec{v}_B = \underline{\begin{bmatrix} 2 \\ -5 \end{bmatrix}}$ ,  $\vec{v}_{B'} = \underline{\begin{bmatrix} -8 \\ -5 \end{bmatrix}}$