姓名: SOLUTION　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________　　**Quiz 10**　　考試日期: 2023/11/29

不可使用手機、計算器，禁止作弊!

1. Consider the set $\mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a, yb]$, and with scalar multiplication defined by $r \otimes [x, y] = [rx, y]$.

   a. Is this set a vector space? **No!**
   Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. What is the zero vector in this vector space? *Hint:* The zero vector will NOT be the vector $[0, 0]$.
   Answer: the zero vector is _____ , for any vectors [x,y], the -[x,y] is _____

Solution：

If the above set is a vector space, then for any vector $[x, y]$, we know that $0 \otimes [x, y]$ must be the zero vector $\vec{0}$.

$$\begin{aligned} \vec{0} = &\ 0 \otimes [1, 1] = [0, 1] \\ = &\ 0 \otimes [1, 2] = [0, 2] \end{aligned}$$

Since $[0, 1] \neq [0, 2]$, it is NOT a vector space.

2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ and $\hat{T} : \mathbb{R}^m \to \mathbb{R}^k$ be linear transformations. Prove directly from its definition that $(T \circ \hat{T}) : \mathbb{R}^n \to \mathbb{R}^k$ is also a linear transformation.

Solution：

2-3 #31.

我上課有證過。