姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________ **Quiz 13** 考試日期: 2023/12/20

不可使用手機、計算器，禁止作弊!

1. Let $T : P_2 \to P_3$ be defined by $T(p(x)) = (x-1)p(x+2)$, the ordered basis for P_2 is $B = (x^2 - x, x^2 + x, 1)$ and the ordered basis for P_3 is $B' = (x^3, x^2, x, 1)$. Fine the standard matrix representation A of T relative to the ordered bases B and B'.

Answer: (a) A $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \\ -4 & -2 & -1 \end{bmatrix}$

(b) Given $p(x)$ so that $p(x)_B = [1, 2, 5]$, find $T(p(x)) =$ $3x^3 + 10x^2 + 6x - 19$

Solution：

$$T(x^2 - x) = (x-1)[(x+2)^2 - (x+2)] = x^3 + 2x^2 - x - 2,$$
$$T(x^2 - x) = (x-1)[(x+2)^2 + (x+2)] = x^3 + 4x^2 + x - 6,$$
$$T(1) = x - 1$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 1 \\ -2 & -6 & -1 \end{bmatrix}, \ rref(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

By the $rref(A)$, we find the $ker(T)_B = \{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \}$, i.e. $ker(T) = \{0x^2 + 0x + 0 = 0\}$

Let $p(x) = 1(x^2 - x) + 2(x^2 + x) + 5(1) = 3x^2 + x + 5$, $p(x)_B = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$.

$$T(p)_{B'} = Ap(x)_B = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 1 \\ -2 & -6 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 6 \\ -19 \end{bmatrix}$$

$T(p) = 3x^3 + 10x^2 + 6x - 19$

2. Circle each of the following True or False. Please give a counterexample （反例）for the false statement and give an explain （解釋）for the true statement.

   (a) True **[False]** The vector space P_{10} of polynomials of degree $\leqslant 10$ is isomorphic to $\mathbb{R}^{10}$.

   Solution：

   P_{10} is isomorphic to $\mathbb{R}^{11}$.

   (b) **[True]** False The function $T_{0'}: V \to V'$ defined by $T_{0'}(\vec{v}) = \vec{0}'$, the zero vector of V', for all $\vec{v}$ in V is a linear transformation.

   Solution：

   For any $\vec{v},\ \vec{u} \in V$,

   $$T_{0'}(\vec{v}) + T_{0'}(\vec{u}) = \vec{0}' + \vec{0}' = \vec{0}' = T_{0'}(\vec{v} + \vec{u})$$

   For any $\vec{v} \in V$, $r \in \mathbb{R}$

   $$rT_{0'}(\vec{v}) = r\vec{0}' = \vec{0}' = T_{0'}(r\vec{v})$$