

不可使用手機、計算器，禁止作弊!

1. Determinant whether the given 4 points lie in a plane in \mathbb{R}^3 . If so, find its area. If not, find its volume.

$$A(1, 1, 1), B(1, -6, 2), C(0, 2, 3), D(-3, 2, 6)$$

Answer: ☒ $ABCD$ are coplanar(共平面), and the area is N/A .

☐ $ABCD$ are NOT coplanar(共平面), and the volume is 4 .

Solution :

$$\overrightarrow{AB} = [0, -7, 1], \overrightarrow{AC} = [-1, 1, 2], \overrightarrow{AD} = [-4, 1, 5]$$

$$\begin{vmatrix} 0 & -7 & 1 \\ -1 & 1 & 2 \\ -4 & 1 & 5 \end{vmatrix} = 24 \neq 0$$

So the points are not coplanar and the volume of the Parallelepiped (平行六面體) formed by coterminous (相鄰邊) edges $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ is 24.

The volume of a tetrahedron (四面體) $ABCD$ formed by coterminous (相鄰邊) edges $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ is

$$\frac{\text{volume of the Parallelepiped}}{6} = \frac{24}{6} = 4$$

2. Let $\vec{b}, \vec{c} \in \mathbb{R}^3$. Show that $\vec{p} = \vec{b} \times \vec{c}$ is perpendicular to both \vec{b} and \vec{c} .

Solution :

Section 4-1 problem 5. 最簡單的做法就是寫下 $\vec{b} \times \vec{c}$ 的定義，然後利用『兩向量內積為零則此二向量垂直』去驗證！