姓名: SOLUTION　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________　　**Quiz 15**　　考試日期: 2024/01/03

不可使用手機、計算器，禁止作弊!

1. Let A be a 7×7 matrix with row vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}, \vec{g}$ and with determinant equal to 10. Find the determinant of the following matrices.
   (a) B is the matrix having row vectors $\vec{a}+6\vec{b}, 3\vec{a}+7\vec{b}-5\vec{c}, \vec{b}, \vec{d}, \vec{e}, \vec{f}, \vec{g}$. $det(B)$= **50**
   (b) C is the matrix having row vectors $\vec{c}+\vec{c}, \vec{b}+\vec{c}, \vec{a}+\vec{c}, \vec{d}, \vec{e}, \vec{f}, \vec{g}$. det($C$)= **-2** .
   (c) Let D is A^{-1}. det(D)= **1/10** .
   (d) Let E is A^T. det(EA)= 100 .
   (e) Let F is $5A$. det(F)= $5^7 \times 10$.

Solution：

Let's present the matrices graphically.

$$A = \begin{bmatrix} - & \vec{a} & - \\ - & \vec{b} & - \\ - & \vec{c} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}, B = \begin{bmatrix} - & \vec{a}+6\vec{b} & - \\ - & 3\vec{a}+7\vec{b}-5\vec{c} & - \\ - & \vec{b} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}, C = \begin{bmatrix} - & \vec{c}+\vec{c} & - \\ - & \vec{b}+\vec{c} & - \\ - & \vec{a}+\vec{c} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}$$

By the Property 1, Property 2, Property 3, Property 4 and Property 5 in Section 4-3

$$\det(B) = \det(\begin{bmatrix} - & \vec{a}+6\vec{b} & - \\ - & 3\vec{a}+7\vec{b}-5\vec{c} & - \\ - & \vec{b} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}) = \det(\begin{bmatrix} - & \vec{a} & - \\ - & -5\vec{c} & - \\ - & \vec{b} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}) = 5\det(A) = 50$$

$$\det(C) = \det(\begin{bmatrix} - & \vec{c}+\vec{c} & - \\ - & \vec{b}+\vec{c} & - \\ - & \vec{a}+\vec{c} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}) = \det(\begin{bmatrix} - & 2\vec{c} & - \\ - & \vec{b} & - \\ - & \vec{a} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}) = -2\det(A) = -20$$

2. Using Cramer's rule to find the component x_2 of the solution for the given linear system.

$$\begin{cases} 6x_1 + x_2 - x_3 = 4 \\ x_1 - x_2 + 5x_4 = -2 \\ -x_1 + 3x_2 + x_3 = 2 \\ x_1 + x_2 - x_3 + 2x_4 = 0 \end{cases}$$

Answer: $x_2 =$ $\underline{\quad 41/59 \quad}$

Solution：

Let $A = \begin{bmatrix} 6 & 1 & -1 & 0 \\ 1 & -1 & 0 & 5 \\ -1 & 3 & 1 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 4 \\ -2 \\ 2 \\ 0 \end{bmatrix}$. Then $B_2 = \begin{bmatrix} 6 & 4 & -1 & 0 \\ 1 & -2 & 0 & 5 \\ -1 & 2 & 1 & 0 \\ 1 & 0 & -1 & 2 \end{bmatrix}$

Thus, $x_2 = \dfrac{\det(B_2)}{\det(A)} = \dfrac{-82}{-118} = \dfrac{41}{59}$.