Quiz 16 練習卷

不可使用手機、計算器，禁止作弊!

1. Determinant whether the given 4 points lie in a plane in $\mathbb{R}^4$. If so, find its area. If not, find its volume.

$$A(2,1,1,1), B(3,1,-6,2), C(5,0,2,3), D(2,-3,2,6)$$

Answer:

✗ $ABCD$ are coplanar(共平面), and the area of the quadrilateral (四邊形) is **N/A** .

✓ $ABCD$ are NOT coplanar, and the volume of the tetrahedron(四面體) is $\frac{\sqrt{20547}}{6}$.

Solution：

$\overrightarrow{AB} = [1,0,-7,1], \overrightarrow{AC} = [3,-1,1,2], \overrightarrow{AD} = [0,-4,1,5]$

$$M = \begin{bmatrix} 1 & 3 & 0 \\ 0 & -1 & -4 \\ -7 & 1 & 1 \\ 1 & 2 & 5 \end{bmatrix}, \quad \det(M^TM) = \begin{vmatrix} 51 & -2 & -2 \\ -2 & 15 & 15 \\ -2 & 15 & 42 \end{vmatrix} = 20547$$

So the points are not coplanar and the volume of the Parallelepiped (平行六面體) formed by coterminous (相鄰邊) edges $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ is $\sqrt{20547}$.

The volume of a tetrahedron (四面體) $ABCD$ formed by coterminous (相鄰邊) edges $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ is

$$\frac{\text{volume of the Parallelepiped}}{6} = \frac{\sqrt{20547}}{6}$$

2. Determinant whether the given 4 points lie in a plane in $\mathbb{R}^4$. If so, find its area. If not, find its volume.

$$A(2,1,1,1), B(3,1,-6,2), C(4,5,-14,-2), D(2,-3,2,6)$$

Answer:

☑ $ABCD$ are coplanar(共平面), and the area of the quadrilateral (四邊形) is $\frac{\sqrt{2138}+\sqrt{8552}}{2}$.

☒ $ABCD$ are NOT coplanar(共平面), and the volume of the tetrahedron (四面體) is **N/A**.

Solution：

$\overrightarrow{AB} = [1,0,-7,1], \overrightarrow{AC} = [2,4,-15,-3], \overrightarrow{AD} = [0,-4,1,5]$

$$M = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 4 & -4 \\ -7 & -15 & 1 \\ 1 & -3 & 5 \end{bmatrix}, \quad \det(M^TM) = \begin{vmatrix} 51 & 104 & -2 \\ 104 & 254 & -46 \\ -2 & -46 & 42 \end{vmatrix} = 0$$

So the points are coplanar.

$$M_1 = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ -7 & -15 \\ 1 & -3 \end{bmatrix}, M_2 = \begin{bmatrix} 2 & 0 \\ 4 & -4 \\ -15 & 1 \\ -3 & 5 \end{bmatrix}$$

The area of triangle ABC is half of the area of the parallelogram(平行四邊形) formed by vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$, which is

$$\frac{\text{area of the parallelogram}}{2} = \frac{\sqrt{\det(M_1^TM_1)}}{2} = \frac{\sqrt{2138}}{2}$$

The area of triangle ABC is half of the area of the parallelogram(平行四邊形) formed by vectors $\overrightarrow{AC}$ and $\overrightarrow{AD}$, which is

$$\frac{\text{area of the parallelogram}}{2} = \frac{\sqrt{\det(M_2^TM_2)}}{2} = \frac{\sqrt{8552}}{2}$$

The area of the quadrilateral (四邊形) $ABCD$ is the sum of the area of triangle ABC and the area of triangle ACD. Thus the area of quadrilateral (四邊形) $ABCD$ is $\frac{\sqrt{2138}+\sqrt{8552}}{2}$.

3. Let A be an $n \times n$ matrix. Prove that

$$\det(adj(A)) = \det(A)^{n-1}$$

Solution：

請見 4-3 節，作業題 38 。