

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Compute the indicated quantity (量), if it is defined.

$$A = \begin{bmatrix} 4 & 1 & -2 \\ 1 & -2 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & 3 \\ -1 & 1 & 2 \end{bmatrix}, C = \begin{bmatrix} 2 & -1 \\ 0 & 5 \\ -1 & 2 \end{bmatrix}$$

$$(a) (3A)(-B) = \underline{\text{undefined}} \quad (b) (3A)(-C) = \underline{\begin{bmatrix} -30 & 9 \\ 3 & 15 \end{bmatrix}}$$

2. For vectors \vec{u} , \vec{v} and \vec{w} in \mathbb{R}^n , prove that $\vec{v} - \vec{w}$ and $\vec{v} + \vec{w}$ are perpendicular(垂直) if and only if $\|\vec{v}\| = \|\vec{w}\|$.

Solution :

1-2 problem 43. 答案曾在 110 學年度第 1 學期的 quiz 2 第二題有提供過。

Circle each of the following True or False and then give a counterexample (反例) for the false statement.

3. True False The magnitude of $\vec{v} + \vec{w}$ must be at least as large as the magnitude of either \vec{v} or \vec{w} in \mathbb{R}^n .

Solution :

Let $\vec{v} = [1, 0]$, $\vec{w} = [-1, 0]$, then $\|\vec{v} + \vec{w}\| = 0 < 1 = \|\vec{v}\| = \|\vec{w}\|$

4. True False There are exactly two unit vectors perpendicular (垂直) to any given nonzero vectors in \mathbb{R}^n .

Solution :

For $n = 3$, $\vec{e}_1 = [1, 0, 0]$ is perpendicular to every vector in $S = \text{sp}(\vec{e}_2, \vec{e}_3) = \text{sp}([0, 1, 0], [0, 0, 1])$.

5. True False For a vector \vec{v} in \mathbb{R}^n , the magnitude(長度) of r times \vec{v} is r times the magnitude of \vec{v} .

Solution :

Should be the "absolute value"(絕對值) of r .