姓名: SOLUTION　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________　　**Quiz 4**　　考試日期: 2023/10/11

不可使用手機、計算器，禁止作弊!

1. Discribe all solutions of a linear system whose corrponding augmented martix can be row-rreduced to the given matrix.

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & 2 & 5 \\ 0 & 0 & 1 & 0 & 3 & 8 \\ 0 & 0 & 0 & 1 & -2 & 4 \end{array}\right]$$

Answer: ☐ the linear system is inconsistent.

☐ the linear system is consistent and the only solution is ________________ .

☑ the linear system is consistent and the solution sets are $\left\{ \begin{bmatrix} 5 \\ 0 \\ 8 \\ 4 \\ 0 \end{bmatrix} + r \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 0 \\ -3 \\ 2 \\ 1 \end{bmatrix} \middle| \; r, s \in \mathbb{R} \right\}$

.

Solution：

Let $x_2 = r, x_5 = s$, then $\begin{cases} 3r + 2s + x_1 = 5 \\ 3s + x_3 = 8 \\ -2s + x_4 = 4 \end{cases}$, we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 - 3r - 2s \\ r \\ 8 - 3s \\ 4 + 2s \\ s \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 8 \\ 4 \\ 0 \end{bmatrix} + r \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 0 \\ -3 \\ 2 \\ 1 \end{bmatrix}$$

2. (a) Find the inverse of the matrix A, if it exists, and (b) express the inverse matrix as a product of elementary matrices. $A = \begin{bmatrix} 6 & 9 \\ -5 & 3 \end{bmatrix}$

Answer: (a) $A^{-1} = \begin{bmatrix} \frac{1}{21} & \frac{-1}{7} \\ \frac{5}{63} & \frac{2}{21} \end{bmatrix}$, (b) $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1/63 \end{bmatrix} \begin{bmatrix} 1 & -1/3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Solution：

$$A^{-1} = \begin{bmatrix} \frac{1}{21} & \frac{-1}{7} \\ \frac{5}{63} & \frac{2}{21} \end{bmatrix}$$

$$\begin{bmatrix} 6 & 9 \\ -5 & 3 \end{bmatrix} \xrightarrow{R_1 \to R_1 + R_2} \begin{bmatrix} 1 & 12 \\ -5 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \to R_2 + 5R_1} \begin{bmatrix} 1 & 12 \\ 0 & 63 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 \to R_1 - \frac{1}{3}R_2} \begin{bmatrix} 1 & 0 \\ 0 & 63 \end{bmatrix} \qquad \begin{bmatrix} 1 & -\frac{1}{3} \\ 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \to \frac{1}{63}R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1/63 \end{bmatrix}$$