Quiz 6

不可使用手機、計算器，禁止作弊!

1. Find 1. the homogeneous solution set and 2. the actually solution set of the following linear system.

$$\begin{cases} 3x_1 + x_2 + x_3 = 8 \\ 6x_1 + 2x_2 + 2x_3 = 16 \\ 9x_1 + 3x_2 + 3x_3 = 24 \end{cases}$$

Answer: 1. $\left\{ r\begin{bmatrix}-1\\1\\0\end{bmatrix} + s\begin{bmatrix}-1\\0\\1\end{bmatrix} \middle| \, r, s \in \mathbb{R} \right\}$, 2. $\left\{ \begin{bmatrix}8\\0\\0\end{bmatrix} + r\begin{bmatrix}-1\\1\\0\end{bmatrix} + s\begin{bmatrix}-1\\0\\1\end{bmatrix} \middle| \, r, s \in \mathbb{R} \right\}$

Solution：

The homogeneous solution is the solution of $\begin{cases} 3x_1 + x_2 + x_3 = 0 \\ 6x_1 + 2x_2 + 2x_3 = 0 \\ 9x_1 + 3x_2 + 3x_3 = 0 \end{cases}$

The corresponding augmented matrix $[A|\vec{b}]$ is

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 8 \\ 6 & 2 & 2 & 16 \\ 9 & 3 & 3 & 24 \end{array}\right] \sim \left[\begin{array}{ccc|c} 3 & 1 & 1 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_2 = r, x_3 = s$, then

$$3x_1 + r + s = 8$$

we have

$$\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}8-r-s\\r\\s\end{bmatrix} = \begin{bmatrix}8\\0\\0\end{bmatrix} + r\begin{bmatrix}-1\\1\\0\end{bmatrix} + s\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

2. Let $\vec{v}$ and $\vec{u}$ be vectors in $\mathbb{R}^n$. Prove the following set equalities by showing that each of the spans is contained in the other.

$$sp(\vec{v}, \vec{u}) = sp(\vec{v}, 2\vec{v} + \vec{u})$$

Solution：

1-6 #45(a)