姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________ **Quiz 7** 考試日期: 2023/11/01

不可使用手機、計算器，禁止作弊!

1. Find a basis for the solution set of the following homogeneous linear system.

$$\begin{cases} x_1 + x_3 - x_4 = 0 \\ x_2 + 2x_3 = 0 \end{cases}$$

Answer: $\left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

Solution：

The corresponding augmented matrix $[A|\vec{b}]$ is

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 2 & 0 & 0 \end{array}\right]$$

Let $x_3 = r, x_4 = s$, then $\begin{cases} r - s + x_1 = 0 \\ 2r + x_2 = 0 \end{cases}$, we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -r+s \\ -2r \\ r \\ s \end{bmatrix} = r\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

2. Prove or disprove the following.
   (a) The sum of two solution vectors of any homogeneous linear system is also a solution vector of the system.

   Solution：

   1-6 #38 (g) or Theorem 1.13 with $r = s = 1$.

(b) If W_1 and W_2 are two subspaces of $\mathbb{R}^n$, then their intersection $W_1 \cap W_2$ is also a subspace.

Solution：

1-6 #44