Quiz 8

不可使用手機、計算器，禁止作弊!

1. Let the subspace $W = sp([1, -3, 2], [2, 1, 3], [1, 11, 0])$ in $\mathbb{R}^3$.

   (a) Find $dim(W) =$ **2** .

   (b) Find a basis for W. Answer: $\{[1, -3, 2], [2, 1, 3]\}$.

   (c) Is $dim(W) = 3$? If not, enlarge the basis you get in (b) to be a basis for $\mathbb{R}^3$.

   Answer: $\{[1, -3, 2], [2, 1, 3], [1, 0, 0]\}$.

   Solution：

   Let $A = \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ -3 & 1 & 11 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 0 & 1 \end{bmatrix}$, and $H = rref(A) = \begin{bmatrix} 1 & 0 & -3 & 0 & -3/11 & 1/11 \\ 0 & 1 & 2 & 0 & 2/11 & 3/11 \\ 0 & 0 & 0 & 1 & -1/11 & -7/11 \end{bmatrix}$

   Since the pivots are in the 1^{st}, 2^{nd} and 4^{th} column of H, we have:

   1. The $dim(W) = 2$.
   2. A basis for W is $\{[1, -3, 2], [2, 1, 3]\}$.
   3. A requested basis for $\mathbb{R}^3$ is $\{[1, -3, 2], [2, 1, 3], [1, 0, 0]\}$.

2. Let $\vec{v}$ and $\vec{w}$ be column vectors in $\mathbb{R}^n$, and let A be an $n \times n$ matrix. Prove that, if $A\vec{v}$ and $A\vec{w}$ are linearly independent, then $\vec{v}$ and $\vec{w}$ are linearly independent.

   Solution：

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