姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________ **Quiz 9** 考試日期: 2023/11/15

不可使用手機、計算器，禁止作弊!

1. Is $T([x,y]) = [5x+4y, x+y, x+1]$ a linear transformation of $\mathbb{R}^2$ to $\mathbb{R}^3$? Why or why not?

Solution：

$$\begin{aligned} T([x,y]+[a,b]) &= T([x+a, y+b]) \\ &= [5(x+a)+4(y+b), (x+a)+(y+b), (x+a)+1] \\ &= [5x+5a+4y+4b, x+a+y+b, x+a+1] \end{aligned}$$

$$\begin{aligned} T(x,y)+T([a,b]) &= [5x+4y, x+y, x+1]+[5a+4b, a+b, a+1] \\ &= [5x+5a+4y+4b, x+a+y+b, x+a+2] \end{aligned}$$

Since $T(x,y)+T([a,b]) \neq T([x,y]+[a,b])$, T is NOT a a linear transformation.

2. Given $A \sim H$, please answer the following questions.

$$A = \begin{bmatrix} 9 & 4 & 0 & 6 & 1 \\ 9 & 0 & 2 & -2 & 5 \\ -6 & 4 & 2 & 4 & -2 \\ -3 & 6 & 1 & 8 & -3 \\ 3 & -4 & 3 & -9 & 6 \end{bmatrix}, H = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is **3** .

(b) Is A invertible? **NO!** .

(c) a basis for the **row space** of A is **[3, 0, 0, 0, 1], [0, 2, 0, 3, -1], [0, 0, 1, -1, 1]** .

(d) a basis for the **column space** of A is $\begin{bmatrix} 9 \\ 9 \\ -6 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 4 \\ 6 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$.

(e) a basis for the **nullspace** of A is $\left\{ \begin{bmatrix} 0 \\ -3/2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/3 \\ 1/2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ **or** $\left\{ \begin{bmatrix} 0 \\ -3 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -6 \\ 0 \\ 6 \end{bmatrix} \right\}$.

Solution：

(a) There's 3 pivots in matrix H.

(b) Pick the rows in **H** which contains a pivot.

(c) Pick the columns in **A** which the corresponding columns in H contains a pivot.

(d) Let $x_4 = r, x_5 = s$. By **H**, $3x_1 + x_5 = 0, 2x_2 + 3x_4 - x_5 = 0, x_3 - x_4 + x_5 = 0$. Thus $x_1 = \frac{-1}{3}s, x_2 = \frac{-3}{2}r + \frac{1}{2}s, x_3 = r - s$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r \begin{bmatrix} 0 \\ -3/2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1/3 \\ 1/2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

3. Let $T([x,y,z]) = [y-z, 2x+z, -x+2y+z]$ an invertible linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$. Find $T^{-1}([5,-3,2])$.

   Answer: $T^{-1}([5,-3,2]) =$ $\frac{-1}{7}[1,16,19]$

   Solution：

   Let A is the standard matrix representation of T.

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix},\ T([x,y,z]) = \left(\begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \right)^T$$

   The standard matrix representation of T^{-1} is A^{-1}

$$A^{-1} = \frac{-1}{7} \begin{bmatrix} -2 & -3 & 1 \\ -3 & -1 & -2 \\ 4 & -1 & -2 \end{bmatrix}$$

$$T^{-1}([5,-3,2]) = (A^{-1} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix})^T = \frac{-1}{7}[1,16,19]$$