姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 12**　　考試日期: 2024/05/22

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Solve the linear system

$$\begin{cases} (5+i)z_1 & +(1-3i)z_2 & = & -4 \\ (2+i)z_1 & +(3-2i)z_2 & = & 2 \end{cases}$$

Answer: $(z_1, z_2) =$ $\left(\frac{-49+35i}{37}, \frac{51+27i}{37}\right)$

Solution：

By problem 2, get A^{-1}

$$A^{-1}\begin{bmatrix} -4 \\ 2 \end{bmatrix} = \begin{bmatrix} (-49+35i)/37 \\ (51+27i)/37 \end{bmatrix}$$

2. find the inverse of A, if

$$A = \begin{bmatrix} (5+i) & (1-3i) \\ (2+i) & (3-2i) \end{bmatrix}$$

Answer: $A^{-1} =$ ____

Solution：

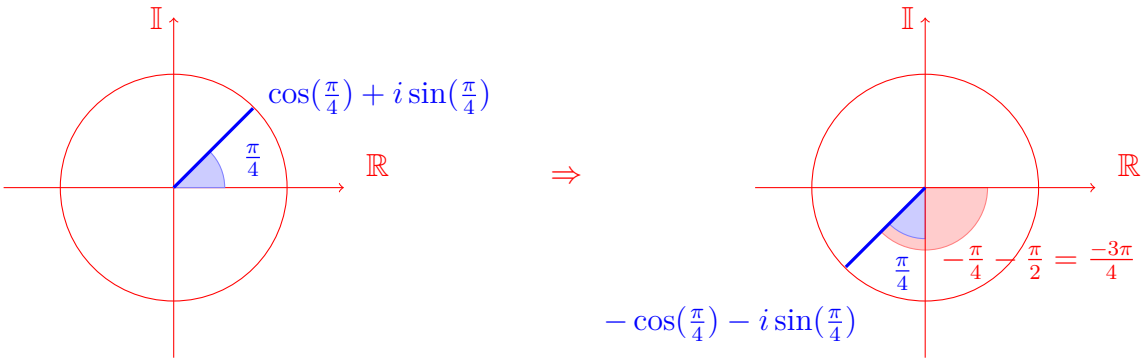
$$A^{-1} = \frac{1}{12-2i}\begin{bmatrix} (3-2i) & -(1-3i) \\ -(2+i) & (5+i) \end{bmatrix} = \frac{1}{74}\begin{bmatrix} (20-9i) & (-9+17i) \\ (-11-8i) & (29+11i) \end{bmatrix}$$

3. Find all the sixth roots of $-2-2i$.

Answer: ____ .

Solution :

$$-2-2i = 2\sqrt{2}\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right) = \sqrt{8}\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right) = \sqrt{8}\left(-\cos(\frac{\pi}{4})-i\sin(\frac{\pi}{4})\right)$$



$$w_k = \sqrt[12]{8}\left(\cos(\frac{-3\pi}{4\times 6}+\frac{2k\pi}{6})+i\sin(\frac{-3\pi}{4\times 6}+\frac{2k\pi}{6})\right),\ k=0,1,2,3,4,5$$
$$= \sqrt[4]{2}\left(\cos(\frac{-\pi}{8}+\frac{k\pi}{3})+i\sin(\frac{-\pi}{8}+\frac{k\pi}{3})\right),\ k=0,1,2,3,4,5$$

4. Prove or disprove that every nonzero complex number has two distinct square roots in $\mathbb{C}$.

Solution :

Section 9-1, problem 17(e).