Quiz 15

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find a 10×10 Jordan canonical form for A, where $(A-3I)$ has rank 8, $(A-3I)^2$ has rank 7, $(A-3I)^3$ has rank 6, $(A-3I)^k$ has rank 5 for $k \geq 4$; $(A+I)^k$ has rank 9 for $k \geq 1$; $(A-2I)$ has rank 8, $(A-2I)^2$ has rank 6, $(A-2I)^k$ has rank 5 for $k \geq 3$.

Answer:

Note that the "rank + nullity = 10", therefore

$(A-3I)$ has rank 8, nullity 2,
$(A-3I)^2$ has rank 7, nullity 3,
$(A-3I)^3$ has rank 6, nullity 4,
$(A-3I)^k$ has rank 5, nullity 5 for $k \geq 4$

$\Rightarrow$ $(A-3I):$ $\vec{e}_1 \to 0$, $\vec{e}_5 \to \vec{e}_4 \to \vec{e}_3 \to \vec{e}_2 \to 0$

$(A+I)^K$ has rank 9, nullity 1 for $k \geq 1$ $\Rightarrow$ $(A+I):$ $\vec{e}_6 \to 0$

$(A-2I)$ has rank 8, nullity 2,
$(A-2I)^2$ has rank 6, nullity 4
$(A-2I)^k$ has rank 5, nullity 5 for $k \geq 3$

$\Rightarrow$ $(A-2I):$ $\vec{e}_8 \to \vec{e}_7 \to 0$, $\vec{e}_{11} \to \vec{e}_{10} \to \vec{e}_9 \to 0$

$$\begin{bmatrix} \boxed{3} & & & & \\ & \boxed{\begin{matrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{matrix}} & & \Large 0 & \\ & & \boxed{-1} & & \\ & \Large 0 & & \boxed{\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}} & \\ & & & & \boxed{\begin{matrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{matrix}} \end{bmatrix}$$

2. Mark all the matrix if it is a Jordan Canonical form and boxed all the Jordan blocks in it.

($\boxed{Yes}$ / No) (a) $\begin{bmatrix} \boxed{2} & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & \boxed{2} \end{bmatrix}$

($\boxed{Yes}$ / No) (b) $\begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \boxed{3} \end{bmatrix}$

($\boxed{Yes}$ / No) (c) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

($\boxed{Yes}$ / No) (d) $\begin{bmatrix} \boxed{3} & 0 & 0 & 0 \\ 0 & \boxed{2} & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & \boxed{4} \end{bmatrix}$

(Yes / $\boxed{No}$) (e) $\begin{bmatrix} i & 1 & 0 & 0 \\ 0 & i & 1 & 0 \\ 0 & 0 & i & 1 \\ 0 & 0 & 0 & -i \end{bmatrix}$ ×