姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________ **Quiz 1** 考試日期: 2024/02/28

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find the characteristic polynomial, the real eigenvalues and a corresponding eigenvector of matrix A.

$$A = \begin{bmatrix} -1 & 0 & 1 \\ -7 & 2 & 5 \\ 3 & 0 & 1 \end{bmatrix}$$

Answer: (a) the characteristic polynomial: $-\lambda^3 + 2\lambda^2 + 4\lambda - 8 = (-2-\lambda)(2-\lambda)^2$.

(b) the eigenvalues and a corresponding eigenvectors: $(2, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}), (-2, \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix})$.

2. Let A is an $n \times n$ invertible matrix and if λ is an eigenvalue of A with $\vec{v}$ as a corresponding eigenvector. Prove that 1. $\lambda \neq 0$ and 2. $1/\lambda$ is an eigenvalue of A^{-1} with $\vec{v}$ as a corresponding eigenvector.

   Solution：

   上課有證，所以我只寫大綱（就是如果考出來你要自己把細節填上）。

   1. Since A is invertible, $\det(A) \neq 0$. Thus $\det(A - 0 \times I) \neq 0$.

   2. Check $A^{-1}\vec{v} = \frac{1}{\lambda}\vec{v}$.

3. Let A is an $n \times n$ matrix and if λ is an eigenvalue of A with $\vec{v}$ as a corresponding eigenvector. What do you know about the eigenvalues and eigenvectors of $A + cI$ for all scalar c

   Solution：

   作業題有下面兩個：

   5-1 23(f) **True** If $\vec{v}$ is an eigenvector of a matrix A, then $\vec{v}$ is an eigenvector of $A + cI$ for all scalar c.

   5-1 23(g) **False** If λ is an eigenvalue of a matrix A, then λ is an eigenvalue of $A + cI$ for all scalar c.

   Let $A\vec{v} = \lambda\vec{v}$, check$(A + cI)\vec{v} = (\lambda + c)\vec{v}$.