姓名: SOLUTION 葉均承 應數一線性代數

學號: ________

Quiz 5

考試日期: 2024/03/27

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Let $W = sp([0,1,1,1],[1,1,1,0],[-1,1,0,1])$ is a subspace of $\mathbb{R}^4$. Using the Gram-Schmidt process to find an orthonormal basis for W and then transform this to an orthonormal basis for $\mathbb{R}^4$. Given $\vec{b} = [-2,3,0,1]$, please find the projection $\vec{b}_W$.

Answer: an orthonormal basis for W is $\{\frac{1}{\sqrt{3}}[0,1,1,1], \frac{1}{\sqrt{15}}[3,1,1,-2], \frac{1}{\sqrt{15}}[-1,3,-2,-1]\}$

$\vec{b}_W = \frac{1}{3}[-5,\ 9,\ -1,\ 4]$

an orthonormal basis for $\mathbb{R}^4$ is $\{\frac{1}{\sqrt{3}}[0,1,1,1], \frac{1}{\sqrt{15}}[3,1,1,-2], \frac{1}{\sqrt{15}}[-1,3,-2,-1], \frac{1}{\sqrt{3}}[1,0,-1,1]\}$

Solution：

Let $\vec{a_1} = [0,1,1,1]$, $\vec{a_2} = [1,1,1,0]$, $\vec{a_3} = [-1,1,0,1]$,

$$\vec{v_1} = [0,1,1,1], \qquad \vec{q_1} = \frac{\vec{v_1}}{\|\vec{v_1}\|} = \frac{1}{\sqrt{3}}[0,1,1,1],$$

$$\vec{v_2} = \vec{a_2} - \frac{\vec{a_2}\cdot\vec{v_1}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1} = \vec{a_2} - \frac{2}{3}\vec{v_1} = \frac{1}{3}[3,1,1,-2], \qquad \vec{q_2} = \frac{\vec{v_2}}{\|\vec{v_2}\|} = \frac{1}{\sqrt{15}}[3,1,1,-2]$$

$$\vec{v_3} = \vec{a_3} - \frac{\vec{a_3}\cdot\vec{v_1}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1} - \frac{\vec{a_3}\cdot\vec{v_2}}{\vec{v_2}\cdot\vec{v_2}}\vec{v_2} = \frac{1}{5}[-1,3,-2,-1], \qquad \vec{q_3} = \frac{\vec{v_3}}{\|\vec{v_3}\|} = \frac{1}{\sqrt{15}}[-1,3,-2,-1],$$

Let $\vec{a}_4 = [1,0,0,0]$, $\vec{a}_5 = [0,1,0,0]$, $\vec{a}_6 = [0,0,1,0]$, $\vec{a}_7 = [0,0,0,1]$

$$\vec{v_4} = \vec{a_4} - \frac{\vec{a_4}\cdot\vec{v_1}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1} - \frac{\vec{a_4}\cdot\vec{v_2}}{\vec{v_2}\cdot\vec{v_2}}\vec{v_2} - \frac{\vec{a_4}\cdot\vec{v_3}}{\vec{v_3}\cdot\vec{v_3}}\vec{v_3} = \frac{1}{3}[1,0,-1,1],$$

$$\vec{q_4} = \frac{\vec{v_4}}{\|\vec{v_4}\|} = \frac{1}{\sqrt{3}}[1,0,-1,1],$$

Method 1:

$$\begin{aligned}\vec{b}_W &= (\vec{b}\cdot\vec{q_1})\,\vec{q_1} + (\vec{b}\cdot\vec{q_2})\,\vec{q_2} + (\vec{b}\cdot\vec{q_3})\,\vec{q_3}\\ &= \frac{4}{\sqrt{3}}\,\vec{q_1} + \frac{-5}{\sqrt{15}}\,\vec{q_2} + \frac{10}{\sqrt{15}}\,\vec{q_3}\\ &= \frac{1}{3}[-5,\ 9,\ -1,\ 4]\end{aligned}$$

Method 2:

$$\vec{b}_W = \vec{b} - \vec{b}_{W^\perp} = \vec{b} - (\vec{b}\cdot\vec{q_4})\vec{q_4} = \vec{b} + \vec{q_4} = \frac{1}{3}[-5,\ 9,\ -1,\ 4]$$