姓名: SOLUTION　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________　　　　**Quiz 8**　　　　考試日期: 2024/04/24

不可使用手機、計算器，禁止作弊!

1. Find the projection matrix for the plane $2x + y - z = 0$ and then find the projection of $[2, 1, 3]$ on the plane.

Answer: $P = \frac{1}{6}\begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix}$, $\vec{b}_W = \frac{1}{6}\begin{bmatrix} 8 \\ 4 \\ 20 \end{bmatrix}$

Solution：

(Method from 6.4 example 3)

Pick $\vec{a}_1 = [1, -2, 0]^T, \vec{a}_2 = [0, 1, 1]^T$ such that $W = sp(\vec{a}_1, \vec{a}_2)$.

$$A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 0 & 1 \end{bmatrix}, (A^T A)^{-1} = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$$

The projection matrix P is

$$P = A(A^T A)^{-1} A^T = \frac{1}{6}\begin{bmatrix} 1 & 0 \\ -2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix}$$

$$\vec{b}_W = P\vec{b} = \frac{1}{6}\begin{bmatrix} 2 & -2 & 2 \\ -2 & 5 & 1 \\ 2 & 1 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 8 \\ 4 \\ 20 \end{bmatrix}$$

2. Find the lease squares solution of the given overdetermined system $A\vec{x} = \vec{b}$.

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

Answer: the lease squares solution is $\begin{bmatrix} -0.2 \\ 0.6 \end{bmatrix}$

Solution：

$\boxed{Method 1}$

Let

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}, \vec{b} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

Then the lease squares solution $\vec{x}' = (A^T A)^{-1} A^T \vec{b} = \begin{bmatrix} -4/3 \\ 2 \end{bmatrix}$

$\boxed{Check}$

Let $\vec{v} = A\vec{x}' - \vec{b} = \frac{1}{3} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$, $\vec{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $\vec{a}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$. Then $\vec{a}_1 \cdot \vec{v} = \vec{a}_2 \cdot \vec{v} = 0$.

$\boxed{Method 2}$

The problem can be written as:

Given $\vec{b} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, $\vec{a}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $\vec{a}_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$. Let $W = sp(\vec{a}_1, \vec{a}_2)$ and $\vec{b}_W$ is the projection of $\vec{b}$ onto W. Find $\vec{r}$ so that $A\vec{r} = \vec{b}_W$.

Then the problem will be the same as the problem 1 in Quiz 4.