姓名: SOLUTION　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________

Quiz 9

考試日期: 2024/05/01

不可使用手機、計算器，禁止作弊!

1. Find the change-of-coordinates matrix from B to B' and from B' to B, indicate which is which, and use it to find the coordinate vector $\vec{v}_{B'}$ with

$$B = ([1,2],[3,4]), \quad B' = ([1,1],[1,-1]), \quad \vec{v}_B = [4,9]$$

Answer: $C_{BB'}=$ $\frac{1}{2}\begin{bmatrix} 3 & 7 \\ -1 & -1 \end{bmatrix}$, $C_{B'B}=$ $\frac{1}{2}\begin{bmatrix} -1 & -7 \\ 1 & 3 \end{bmatrix}$, $\vec{v}_{B'}=$ $\frac{1}{2}\begin{bmatrix} 75 \\ -13 \end{bmatrix}$, $\vec{v}=$ $\begin{bmatrix} 31 \\ 44 \end{bmatrix}$.

Solution：

$$M_B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, \quad M_{B'} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Using

$$[M_{B'} \mid M_B] = [I \mid C_{B,B'}]$$

or

$$C_{B,B'} = M_{B'}^{-1} M_B = \frac{1}{2}\begin{bmatrix} 3 & 7 \\ -1 & -1 \end{bmatrix}$$

$$C_{B',B} = C_{B,B'}^{-1} = \frac{1}{2}\begin{bmatrix} -1 & -7 \\ 1 & 3 \end{bmatrix}$$

$$\vec{v} = M_B \vec{v}_B = \begin{bmatrix} 31 \\ 44 \end{bmatrix}$$

$$\vec{v}_{B'} = C_{B,B'} \vec{v}_B = \frac{1}{2}\begin{bmatrix} 75 \\ -13 \end{bmatrix}$$

2. Let B, B' and B'' be ordered bases for $\mathbb{R}^n$. Find the change-of-coordinates matrix from B to B'' in terms of $C_{B,B'}$ and $C_{B',B''}$. [Hint: For a vector $\vec{v}$ in $\mathbb{R}^n$, with matrix times $\vec{v}_B$ gives $\vec{v}_{B''}$?] (注意這不是填充題，猜答案沒有分)

   Solution：

   7-1 #25

$$C_{B,B''} = C_{B',B''}C_{B,B'}$$

3. Prove that if B and B' are orthonormal bases, then $C_{B,B'}$ is an orthogonal matrix.

   Solution：

   7-1 #23(c)