姓名: SOLUTION　　　　　　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ______________　　　　**Quiz 10**　　　　考試日期: 2024/11/27

不可使用手機、計算器，禁止作弊!

1. Let $P$ is the vector space of polynomials. Prove that $sp(1, x) = sp(1 + 2x, 3x)$ [HINT: Show that each of these subspaces is a subset of the other.]

   **Solution：**

   3-2, problem 8.

2. Determine whether the given set $S$ of vectors is dependent of independent. Then reduce the given set to be a basis for $sp(S)$.

   $$S = \{1, e^x + e^{-x}, e^x - e^{-x}\} \text{ is a subset in a vector space } P.$$

   Answer: Is $S$ independent: ( $\boxed{Yes}$ / No ) .

   The basis for $sp(S)$ is $\{1, e^x + e^{-x}, e^x - e^{-x}\}$

   **Solution：**

   3-2, problem 19. (Similar with 3-2 example 3.)

   Suppose $r_1 \times 1 + r_2 \times (e^x + e^{-x}) + r_3 \times (e^x - e^{-x}) = 0$

   Differentiating the equation twice, we have:

   $$r_2 \times (e^x + e^{-x}) + r_3 \times (e^x - e^{-x}) = 0$$

   Thus, we have $r_1 = 0$.

   Differentiating the equation once, we have:

   $$r_2 \times (e^x - e^{-x}) + r_3 \times (e^x + e^{-x}) = 0$$

   Substituting $x = 0$ into the above two equations, we have

   $$2r_2 = 0, \ 2r_3 = 0$$

   Therefore, $r_1 = r_2 = r_3 = 0$. $S$ is independent and $S$ is a basis for $sp(S)$.

Circle each of the following True or False. Please give a counterexample （反例）for the false statement and give an explain （解釋）for the true statement.

3. True **False** If $S$ is independent, each vector in $V$ can be expressed uniquely as a linear combination of vectors in $S$.

**Solution：**

3-2, 25(g)

4. **True** False Every vector space with a nonzero vector has at least two distinct subspaces.

**Solution：**

3-2, 25(b)

5. True **False** If $\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_n\}$ generates $V$, then each $\vec{v} \in V$ is a unique linear combination in this set.

**Solution：**

3-2, 26(c)