姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________ **Quiz 11** 考試日期: 2024/12/04

不可使用手機、計算器，禁止作弊!

1. For each integer n, we have $\{1, \sin(x), \sin(2x), ..., \sin(nx)\}$ is an linear independent subset of the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$. Find a basis for the subspace of F spanned by the functions

$$\begin{aligned} f_1(x) &= 3 - \sin(x) + 3\sin(2x) \\ f_2(x) &= 1 + 2\sin(x) + 4\sin(2x) \\ f_3(x) &= -1 + 5\sin(x) + 5\sin(2x) \\ f_4(x) &= 1 + 3\sin(2x) \end{aligned}$$

Answer: the basis is $\{f_1(x), f_2(x), f_4(x)\}$

Solution：

Similar with 3-3 example 4.

$$rref(\begin{bmatrix} 3 & 1 & -1 & 1 \\ -1 & 2 & 5 & 0 \\ 3 & 4 & 5 & 3 \end{bmatrix}) = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Let T_1, T_2 are both a linear transformation from V to V' and let $(T_1 + T_2) : V \to V'$ be defined by

$$(T_1 + T_2)(\vec{v}) = T_1(\vec{v}) + T_2(\vec{v})$$

for each $\vec{v}$ in V. Prove that $T_1 + T_2$ is also a linear transformation.

Solution：

3-2 problem 43.