姓名: SOLUTION 葉均承 應數一線性代數

學號: ______________ **Quiz 12** 考試日期: 2024/12/11

不可使用手機、計算器，禁止作弊!

1. Determinant whether the given 4 points lie in a plane in $\mathbb{R}^3$. If so, find its area. If not, find its volume.

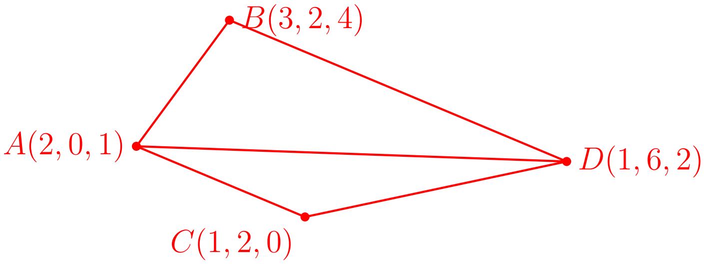
$$A(2,0,1), B(3,2,4), C(1,2,0), D(1,6,2)$$

Answer: ☑ $ABCD$ are coplanar(共平面), and the area is $\sqrt{336}+\sqrt{84}$.

☒ $ABCD$ are NOT coplanar(共平面), and the volume is **N/A** .

Solution：

Similar with 112-1 quiz 14. The volume is 0, thus they are coplanar.



The area of a triangle in $\mathbb{R}^3$ can be determined using the method outlined in Example 5 of Section 4-1.

The area of ABD, using $\overrightarrow{AB}=[3-1,2-0,4-1]=[1,2,3]$, $\overrightarrow{AD}=[1-2,6-0,2-1]=[-1,6,1]$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ -1 & 6 & 1 \end{vmatrix} = [-16,-4,8], \quad \Rightarrow \|[-16,-4,8]\| = \sqrt{336}$$

The area of ACD, using $\overrightarrow{AC}=[1-2,2-0,0-1]=[-1,2,-1]$, $\overrightarrow{AD}=[1-2,6-0,2-1]=[-1,6,1]$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & -1 \\ -1 & 6 & 1 \end{vmatrix} = [8,2,-4], \quad \Rightarrow \|[8,2,-4]\| = \sqrt{84}$$

2. Let $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^3$. Show that $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

Solution：

Section 4-1 problem 59. 用定義驗證