

不可使用手機、計算器，禁止作弊!

1. Let A be a 7×7 matrix with row vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}, \vec{g}$ and with determinant equal to 20. Find the determinant of the following matrices.

(a) B is the matrix having row vectors $(\vec{a} + 5\vec{g}), (3\vec{a} + 7\vec{b} + 5\vec{c}), \vec{b}, (\vec{d} - 9\vec{e}), \vec{e}, \vec{f}, \vec{g}$.

$$\det(B) = \underline{-100}$$

(b) C is the matrix having row vectors $(\vec{c} + \vec{c}), (\vec{b} + \vec{c}), (\vec{a} + \vec{c}), (\vec{d} - \vec{f}), \vec{e}, \vec{f}, \vec{g}$.

$$\det(C) = \underline{-40}.$$

(c) Let D is A^{-1} . $\det(D) = \underline{1/20}$.

(d) Let E is A^T . $\det(EA) = \underline{400}$.

(e) Let F is $5A$. $\det(F) = \underline{5^7 \times 20}$.

Solution :

Let's present the matrices graphically.

$$A = \begin{bmatrix} - & \vec{a} & - \\ - & \vec{b} & - \\ - & \vec{c} & - \\ - & \vec{d} & - \\ - & \vec{e} & - \\ - & \vec{f} & - \\ - & \vec{g} & - \end{bmatrix}$$

Similar to 112-1 quiz 15.

2. Prove or disprove the following:

- (a) True **False** The product of a square matrix and its adjoint is the identity matrix.

Solution :

4-3 problem 35 (g)

- (b) **True** False Let A be an $n \times n$ matrix, then $\det(\text{adj}(A)) = \det(A)^{n-1}$.

Solution :

4-3 problem 38