姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 14**　　考試日期: 2024/12/25

不可使用手機、計算器，禁止作弊!

1. Determinant whether the given 4 points lie in a plane in $\mathbb{R}^4$. If so, find its area. If not, find its volume.

$$A(1,0,0,1), B(-1,2,0,1), C(3,0,1,1), D(-1,4,0,1)$$

Answer:

☒ $ABCD$ are coplanar(共平面), and the area of the quadrilateral (四邊形) is **N/A** .

☑ $ABCD$ are NOT coplanar, and the volume of the tetrahedron(四面體) is $\frac{2}{3}$ .

**Solution：**

See 112-1 quiz 16 problem 1.

2. Let $G$ be the tetrahedron determined in previous problem. Let $T : \mathbb{R}^4 \to \mathbb{R}^5$ be given by $T([x, y, z, w]) = [2x + 3y, x - y, 2y + w, z + w, x - w]$. Find the volume of the image of G in $\mathbb{R}^5$ under the transformation $T$.

   Answer: $\frac{2\sqrt{60}}{3} = \frac{4\sqrt{15}}{3}$

   **Solution：**

   Similar with example 7 in section 4-4.

   By Theorem 4.9, we have the result should be $\sqrt{\det(A^T A)} \cdot V$ where $V$ is the volume of $G$, which is $2/3$ and $A$ is the standard matrix representation of $T$

$$A = \begin{bmatrix} 2 & 3 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

$$\det(A^T A) = \begin{vmatrix} 6 & 5 & 0 & -1 \\ 5 & 14 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ -1 & 2 & 1 & 3 \end{vmatrix} = 60$$