姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________ **Quiz 4** 考試日期: 2024/10/09

不可使用手機、計算器，禁止作弊!

1. Discribe all solutions of a linear system whose corrponding augmented martix can be row-rreduced to the given matrix.

$$\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 0 & 2 & 5 \\ 0 & 1 & 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 1 & -1 & 9 \end{array}\right]$$

Answer: ✗ the linear system is inconsistent.

✗ the linear system is consistent and the only solution is ________________ .

✓ the linear system is consistent and the solution sets are $\left\{ \begin{bmatrix} 5 \\ 7 \\ 0 \\ 9 \\ 0 \end{bmatrix} + r \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ -3 \\ 0 \\ -1 \\ 1 \end{bmatrix} \middle| \, r, s \in \mathbb{R} \right\}$.

Solution：

Let $x_3 = r, x_5 = s$, then $\begin{cases} -2r + 2s + x_1 = 5 \\ 3s + x_2 = 7 \\ -s + x_4 = 9 \end{cases}$, we have

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 5 + 2r - 2s \\ 7 - 3s \\ r \\ 9 - s \\ s \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 0 \\ 9 \\ 0 \end{bmatrix} + r \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ -3 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

2. (a) Find the inverse of the matrix A, if it exists, and (b) express the inverse matrix as a product of elementary matrices. $A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$

Answer: (a) $A^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 5 \\ 3 & -2 \end{bmatrix}$, (b) **不唯一**

Solution：

$$A^{-1} = \begin{bmatrix} \frac{1}{21} & \frac{-1}{7} \\ \frac{5}{63} & \frac{2}{21} \end{bmatrix} = \begin{bmatrix} 1 & -5/2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -2/13 \end{bmatrix}\begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -3/2 & 1 \end{bmatrix}$$

3. Let $W = \{[x, y, z] \mid x, y, z \in \mathbb{R} \text{ and } z = 2x + y + 1\}$. Determine whether the set W is a subspace of $\mathbb{R}^3$. Please give reasons to support your answer.

Answer: (Yes / $\boxed{No}$) , and write your reason below.

Solution：

subset:
All the elements in W form as $[x, y, 2x + y + 1]$ that are vectors in $\mathbb{R}^2$.

closed under vector addition:
For any $\vec{v}, \vec{u} \in W$. Let $\vec{v} = [x, y, 2x + y + 1], \vec{u} = [a, b, 2a + b + 1]$, $x, y, a, b \in \mathbb{R}$.
We have$\vec{v} + \vec{u} = [x, y, 2x + y + 1] + [a, b, 2a + b + 1] = [(x + a), (y + b), (x + a) + 2(y + b) + 2]$.
Since the 3^{rd} component $((x + a) + 2(y + b) + 2)$ is NOT in the form $(2(x + a) + (y + b) + 1)$, we have $(\vec{v} + \vec{u}) \notin W$.

Hence W is NOT a subspace of $\mathbb{R}^2$