姓名: SOLUTION 葉均承 應數一線性代數

學號: ____________

Quiz 6

考試日期: 2024/10/23

不可使用手機、計算器，禁止作弊!

1. Given vectors $\vec{w}_1 = [1, 2, 3, -1]$, $\vec{w}_2 = [-2, -3, -5, 1]$ and $\vec{w}_3 = [-1, -3, -4, 2]$.
   (a) Determine whether the vectors $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ form a basis for the $sp(\vec{w}_1, \vec{w}_2, \vec{w}_3)$. (Yes / **No**)
   (b) Determine whether the vectors $\{\vec{w}_1, \vec{w}_2\}$ form a basis for the $sp(\vec{w}_1, \vec{w}_2)$. (**Yes** / No) .
   (c) Determine whether the vectors $\{\vec{w}_1, \vec{w}_3\}$ form a basis for the $sp(\vec{w}_1, \vec{w}_3)$. (**Yes** / No) .
   (d) Is $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ a linear independent set? (Yes / **No**) .
   (e) Is $\{\vec{w}_1, \vec{w}_2\}$ a linear independent set?? (**Yes** / No) .
   (f) Is $\{\vec{w}_1, \vec{w}_3\}$ a linear independent set?? (**Yes** / No) .

   p.s. 記得每小題要分開給理由！!

Solution：

1-6 example 5 (我沒改數字)

(1)

$$\begin{bmatrix} 1 & -2 & -1 \\ 2 & -3 & -3 \\ 3 & -5 & -4 \\ -1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

, Since not every column has piovts, (a)(d) is NO!

(2)

$$\begin{bmatrix} 1 & -2 \\ 2 & -3 \\ 3 & -5 \\ -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

, Since every column has piovts, (b)(e) is yes!

(3)

$$\begin{bmatrix} 1 & -1 \\ 2 & -3 \\ 3 & -4 \\ -1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

, Since every column has piovts, (c)(f) is yes!

p.s. 其實你可以看出來後面兩個部分的矩陣，其實只是第一部分的那個矩陣的某兩個 column 而已，所以其實可以只要算一次，後面直接從第一部分的結果擷取就好。

2. Let $\vec{v}_1, \vec{v}_2, \vec{v}_3$ be vectors in $\mathbb{R}^n$. Prove that $\vec{w}_1 = 2\vec{v}_1 + 3\vec{v}_2, \vec{w}_2 = \vec{v}_2 - 2\vec{v}_3$ and $\vec{w}_3 = -\vec{v}_1 - 3\vec{v}_3$ are linearly dependent.

Solution：

2-1 #31

Since

$$\vec{w}_1 - 3\vec{w}_2 + 2\vec{w}_3 = \vec{0},$$

we know that $\{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is a linearly dependent set.