姓名:_____ 葉均承 應數一線性代數

學號: _____ **Quiz 6** 考試日期: 2024/10/23

不可使用手機、計算器，禁止作弊!

1. Given $A \sim H$, please answer the following questions.

$$A = \begin{bmatrix} 9 & 4 & 0 & 6 & 1 \\ 9 & 0 & 2 & -2 & 5 \\ -6 & 4 & 2 & 4 & -2 \\ -3 & 6 & 1 & 8 & -3 \\ 3 & -4 & 3 & -9 & 6 \end{bmatrix}, H = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is _____.

(b) Is A invertible? _____.

(c) a basis for the **row space** of A is _____.

(d) a basis for the **column space** of A is _____ .

(e) a basis for the **nullspace** of A is _____.

2. Prove or disprove (反證) the following statement.

   (a) The column space of AC is contained in the column space of A.

   (b) $rank(AC) \leq rank(A)$.

   (c) The column space of AC is contained in the column space of C.

   (d) $rank(AC) \leq rank(C)$.

   (e) Let $\vec{v}, \vec{w}$ be column vectors in $\mathbb{R}^n$ and let A be an $n \times n$ matrix. If $A\vec{v}$ and $A\vec{w}$ are linearly independent, then $\vec{v}$ and $\vec{w}$ are linearly independent

   (f) Let $\vec{v}, \vec{w}$ be column vectors in $\mathbb{R}^n$ and let A be an $n \times n$ matrix. If $\vec{v}$ and $\vec{w}$ are linearly independent, then $A\vec{v}$ and $A\vec{w}$ are linearly independent

3. Find all scalars s if any exist, such that $[1, 0, 1], [2, s, 3], [1, -2s, 0]$ are linearly independent.