學號: ______________

Quiz 6

考試日期: 2024/10/23

不可使用手機、計算器，禁止作弊!

1. Given $A \sim H$, please answer the following questions.

$$A = \begin{bmatrix} 9 & 4 & 0 & 6 & 1 \\ 9 & 0 & 2 & -2 & 5 \\ -6 & 4 & 2 & 4 & -2 \\ -3 & 6 & 1 & 8 & -3 \\ 3 & -4 & 3 & -9 & 6 \end{bmatrix}, H = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is **3** .

(b) Is A invertible? **NO!** .

(c) a basis for the **row space** of A is **[3, 0, 0, 0, 1], [0, 2, 0, 3, -1], [0, 0, 1, -1, 1]** .

(d) a basis for the **column space** of A is $\begin{bmatrix} 9 \\ 9 \\ -6 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 4 \\ 6 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$.

(e) a basis for the **nullspace** of A is $\left\{ \begin{bmatrix} 0 \\ -3/2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1/3 \\ 1/2 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$ **or** $\left\{ \begin{bmatrix} 0 \\ -3 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -6 \\ 0 \\ 6 \end{bmatrix} \right\}$.

Solution：

(a) There's 3 pivots in matrix H.

(b) Pick the rows in **H** which contains a pivot.

(c) Pick the columns in **A** which the corresponding columns in H contains a pivot.

(d) Let $x_4 = r, x_5 = s$. By **H**, $3x_1 + x_5 = 0, 2x_2 + 3x_4 - x_5 = 0, x_3 - x_4 + x_5 = 0$. Thus $x_1 = \frac{-1}{3}s, x_2 = \frac{-3}{2}r + \frac{1}{2}s, x_3 = r - s$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r \begin{bmatrix} 0 \\ -3/2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1/3 \\ 1/2 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

2. Prove or disprove (反證) the following statement.

   (a) The column space of AC is contained in the column space of A.

   Solution：

   It is true! 2-2, problem 14.

   (b) $rank(AC) \leq rank(A)$.

   Solution：

   It is true! 2-2, problem 18.

   (c) The column space of AC is contained in the column space of C.

   Solution：

   It is false! 2-2, problem 15.

   (d) $rank(AC) \leq rank(C)$.

   Solution：

   It is true! 2-2, problem 20.

   (e) Let $\vec{v}, \vec{w}$ be column vectors in $\mathbb{R}^n$ and let A be an $n \times n$ matrix. If $A\vec{v}$ and $A\vec{w}$ are linearly independent, then $\vec{v}$ and $\vec{w}$ are linearly independent

   Solution：

   It is true! 2-1, problem 36.

   (f) Let $\vec{v}, \vec{w}$ be column vectors in $\mathbb{R}^n$ and let A be an $n \times n$ matrix. If $\vec{v}$ and $\vec{w}$ are linearly independent, then $A\vec{v}$ and $A\vec{w}$ are linearly independent

   Solution：

   It is false! Compare with 2-1, problem 34, the hypothesis missing the condition that A is invertible.

3. Find all scalars s if any exist, such that $[1, 0, 1], [2, s, 3], [1, -2s, 0]$ are linearly independent.

   Solution：

   Similar with 2-1 problem 33. For all $s \neq 0$, $[1, 0, 1], [2, s, 3], [1, -2s, 0]$ are linearly independent.