姓名: SOLUTION 　　　　　　　　葉均承 　　應數一線性代數

學號: ________ 

# Quiz 8

考試日期: 2024/11/13

不可使用手機、計算器，禁止作弊!

1. Given a linear transformation such that $T([-1,2]) = [1,0,0]$ and $T([3,1]) = [0,1,2]$. Please find the standard matrix representation of $T$ and $T([1,10])$.

   Answer: the s.m.r of $T$ is $\frac{1}{7}\begin{bmatrix} -1 & 3 \\ 2 & 1 \\ 4 & 2 \end{bmatrix}$ and $T([1,10]) =$ $\frac{1}{7}[29,12,24]$ .

   **Solution：**

$$\left[\begin{array}{cc|cc} -1 & 3 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & -1/7 & 3/7 \\ 0 & 1 & 2/7 & 1/7 \end{array}\right]$$

   Thus

$$T([1,0]) = \frac{-1}{7}T([-1,2]) + \frac{2}{7}T([3,1]) = \frac{-1}{7}[1,0,0] + \frac{2}{7}[0,1,2] = \frac{1}{7}[-1,2,4]$$

$$T([0,1]) = \frac{3}{7}T([-1,2]) + \frac{1}{7}T([3,1]) = \frac{3}{7}[1,0,0] + \frac{1}{7}[0,1,2] = \frac{1}{7}[3,1,2]$$

$$A = \begin{bmatrix} | & | \\ T(\vec{e}_1) & T(\vec{e}_2) \\ | & | \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -1 & 3 \\ 2 & 1 \\ 4 & 2 \end{bmatrix}$$

$$T([1,10]) = T([1,0]) + 10T([0,1]) = \frac{1}{7}[-1,2,4] + \frac{10}{7}[3,1,2] = \frac{1}{7}[29,12,24]$$

2. Prove or disprove (反證) the following statement.

   (a) If $T$ and $\tilde{T}$ are different linear transformations mapping $\mathbb{R}^n$ into $\mathbb{R}^m$, then we may have $T(\vec{e}_i) = \tilde{T}(\vec{e}_j)$ for some standard basis vector $\vec{e}_i$ of $\mathbb{R}^n$.

   **Solution：**

   It is true! 2-3, problem 29h.

   (b) If $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \cdots \vec{b}_n\}$ is a basis for $\mathbb{R}^n$ and $T$ and $\tilde{T}$ are linear transformations mapping $\mathbb{R}^n$ into $\mathbb{R}^m$, then $T(\vec{x}) = \tilde{T}(\vec{x})$ for all $\vec{x} \in \mathbb{R}^n$ if and only if $T(\vec{b}_i) = \tilde{T}(\vec{b}_i)$ for $i = 1, 2, \cdots, n$.

   **Solution：**

   It is true! 2-3, problem 29j.