姓名: SOLUTION 葉均承 應數一線性代數

學號: ______

Quiz 10

考試日期: 2025/05/07

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Given an linear transformation T, where T is defined on $\mathbb{R}^3$ by $T([x,y,z]) = [3x+2y, 2x, x+2z]$. Find $T^{20}([1,1,1])$.

   Answer: $T^{20}([1,1,1]) =$ $[-3+18\times 4^{20},\ 6+9\times 4^{20},\ 1+5\times 2^{20}+9\times 4^{20}]$.

Solution：

計算的時候先擺成 column vector，寫答案的時候記得要改回 row vector。

Let A be the standard matrix representation of T.

$$T(\begin{bmatrix} x\\ y\\ z\end{bmatrix}) = \begin{bmatrix} 3x+2y\\ 2x\\ x+2z\end{bmatrix} = \begin{bmatrix} 3 & 2 & 0\\ 2 & 0 & 0\\ 1 & 0 & 2\end{bmatrix}\begin{bmatrix} x\\ y\\ z\end{bmatrix} = A\begin{bmatrix} x\\ y\\ z\end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 0 & 2\\ 6 & 0 & 1\\ 1 & 1 & 1\end{bmatrix},\ D = \begin{bmatrix} -1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 4\end{bmatrix},\ A = CDC^{-1}$$

Method 1 7-2 example 5. (讓大家回去讀的，我連數字都沒換，只改了次數)

Method 2

$$C^{-1} = \frac{1}{15}\begin{bmatrix} -1 & 2 & 0\\ -5 & -5 & 15\\ 6 & 3 & 0\end{bmatrix}$$

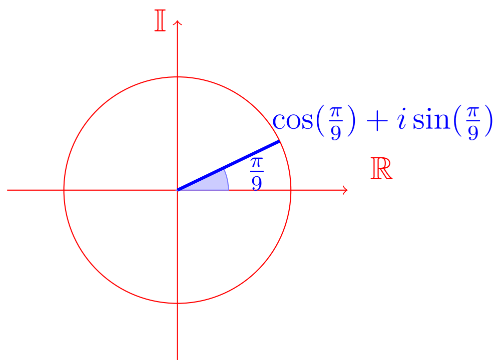
$$\begin{aligned} T^{20}(\begin{bmatrix} x\\ y\\ z\end{bmatrix}) &= A^{20}\begin{bmatrix} x\\ y\\ z\end{bmatrix} = CD^{20}C^{-1}\begin{bmatrix} x\\ y\\ z\end{bmatrix} \\ &= \begin{bmatrix} 3+12\times 4^{20} & -6+6\times 4^{20} & 0\\ -6+6\times 4^{20} & 12+3\times 4^{20} & 0\\ -1-5\times 2^{20}+6\times 4^{20} & 2-5\times 2^{20}+3\times 4^{20} & 15\times 2^{20}\end{bmatrix}\begin{bmatrix} x\\ y\\ z\end{bmatrix}\end{aligned}$$

$$T^{20}(\begin{bmatrix} 1\\ 1\\ 1\end{bmatrix}) = \begin{bmatrix} 3+12\times 4^{20}-6+6\times 4^{20}+0\\ -6+6\times 4^{20}+12+3\times 4^{20}+0\\ -1-5\times 2^{20}+6\times 4^{20}+2-5\times 2^{20}+3\times 4^{20}+15\times 2^{20}\end{bmatrix} = \begin{bmatrix} -3+18\times 4^{20}\\ 6+9\times 4^{20}\\ 1+5\times 2^{20}+9\times 4^{20}\end{bmatrix}$$

2. Find the modulus and principal argument of $3(\sin(\frac{\pi}{9}) - i\cos(\frac{\pi}{9}))$.

   Answer: modulus = **3** , principal argument= $\frac{-7\pi}{18}$.

   Solution：



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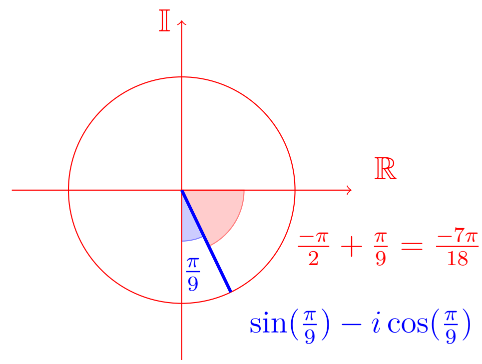
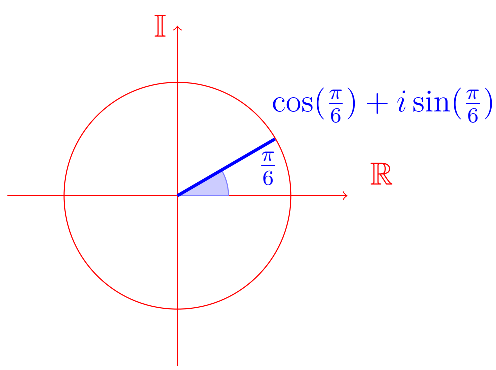


3. Find all the sixth roots of $-2\sqrt{3} - 2i$.

   Answer: $\sqrt[3]{2}\ \left(\cos(\frac{-5\pi}{36} + \frac{k\pi}{3}) + i\sin(\frac{-5\pi}{36} + \frac{k\pi}{3})\right),\ k = 0, 1, 2, 3, 4, 5$.

   Solution：

$$-2\sqrt{3} - 2i = 4\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = 4\left(-\cos(\frac{\pi}{6}) - i\sin(\frac{\pi}{6})\right) = 4\left(\cos(\frac{-5\pi}{6}) + i\sin(-5\frac{\pi}{6})\right)$$



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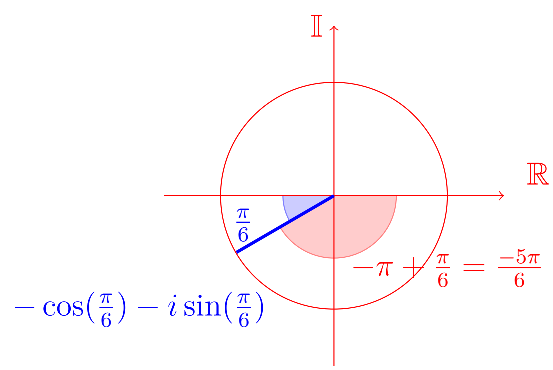

$$\begin{aligned} w_k &= \sqrt[6]{4}\ \left(\cos(\frac{-5\pi}{6\times 6} + \frac{2k\pi}{6}) + i\sin(\frac{-5\pi}{5\times 6} + \frac{2k\pi}{6})\right),\ k = 0, 1, 2, 3, 4, 5 \\ &= \sqrt[3]{2}\ \left(\cos(\frac{-5\pi}{36} + \frac{k\pi}{3}) + i\sin(\frac{-5\pi}{36} + \frac{k\pi}{3})\right),\ k = 0, 1, 2, 3, 4, 5 \end{aligned}$$