姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 11**　　考試日期: 2025/05/14

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find a vector perpendicular to both $[1-i,\ 2,\ 2+i], [i,\ 1+i,\ -1]$.

   Answer: $[-3+3i,\ -i,\ 2+2i]$

   Solution：

   $\boxed{Method\ 1}$ Gram-Schmitt process

   $\boxed{Method\ 2}$ Cross product

2. Find the dot product of $[1-i,\ 2,\ 2+i]$ and $[i,\ 1+i,\ -1]$.

   Answer: $-1+4i$

   Solution：

   $< [1-i, 2, 2+i], [i, 1+i, -1] >= -1+4i$

3. If A is an $n \times n$ normal matrix (i.e. $A^*A = AA^*$), prove that for any $\vec{z} \in \mathbb{C}^n$, $\|A\vec{z}\| = \|A^*\vec{z}\|$.

Solution：

Section 9.2, problem 44.

Hint: $\|A\vec{z}\|^2 =< A\vec{z}, A\vec{z} >= (A\vec{z})^*(A\vec{z})$