姓名: SOLUTION 葉均承 應數一線性代數

學號: ________________ **Quiz 1** 考試日期: 2025/02/26

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find the characteristic polynomial, the real eigenvalues and a corresponding eigenvector of matrix A.

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -8 & 4 \\ 1 & 0 & -4 \end{bmatrix}$$

Answer: (a) the characteristic polynomial: $\lambda^3 + 12\lambda^2 + 32\lambda = \lambda(-8-\lambda)(-4-\lambda)^2$.

(b) the eigenvalues and a corresponding eigenvectors: $(-8, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}), (-4, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}), (0, \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix})$

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2. The first two terms of the sequences are $a_0 = 0$ and $a_1 = 3$. Subsequent terms are generated using the relation

$$a_k = 5a_{k-1} - a_{k-2},\ \ for\ k \geq 2$$

(a) Write the terms of the sequence through a_4. $a_2 = 15$, $a_3 = 72$, $a_4 = 345$

(b) Find a matrix that can be used to generated the sequences, as the matrix A shown in the class. $A = \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix}$

(c) Using the above matrix to find the a_2 : $\begin{bmatrix} 15 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$, $a_2 = 15$

Solution：

$$\begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix} = \begin{bmatrix} 5a_{k-1} - a_{k-2} \\ a_{k-1} \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_{k-1} \\ a_{k-2} \end{bmatrix}$$

(C)

$$\begin{bmatrix} a_2 \\ a_1 \end{bmatrix} = \begin{bmatrix} 15 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix},\ a_2 = 15$$