1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Given a matrix A and use it to answer the following question. (a) find the eigenvalues and a corresponding eigenvectors of A. (b) find the algebraic multiplicity and the geometric multiplicity of each eigenvalue.

$$A = \begin{bmatrix} -1 & 5 & 1 \\ 0 & 3 & 0 \\ 8 & 1 & 1 \end{bmatrix}$$

Answer: (a) find the algebraic multiplicity and the geometric multiplicity of each eigenvalue.

Answer: 　**for eigenvalue -3, its alg. multiplicity is 1 where its geo. multiplicity is 1.**
　　for eigenvalue 3, its alg. multiplicity is 2 where its geo. multiplicity is 1.

(b) Is A diagonalizable?　(Yes / $\boxed{No}$)　. If so, find an invertible C for (c).
If not, why?　**for eigenvalue 3, its alg. multiplicity is 2 where its geo. multiplicity is 1.**

(c) Find a matrix C and a diagonal matrix D such that $AC = CD$.

Answer: C= $\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 0 \\ 2 & 4 & 4 \end{bmatrix}$, and D= $\begin{bmatrix} -3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

Solution：

Similar with 112-2 quiz 2 problem 1.

2. Please give a counterexample (反例) for each statement to prove it wrong.

(a) Every $n \times n$ matrix is diagonalizable.
Solution：
5-2 # 13 (a)

(b) An $n \times n$ matrix is diagonalizable if and only if it has n distinct eigenvalues.
Solution：
5-2 # 13 (d)

(c) Every invertible matrix is diagonalizable.
Solution：
5-2 # 13 (f)

(d) If an $n \times n$ matrix A is diagonalizable, there is a unique matrix D that is similar to A.
Solution：
5-2 # 13 (i)