姓名: <u>SOLUTION</u>

Quiz 6

考試日期: 2025/04/02

1. 請框出答案. 2. 不可使用手機、計算器,禁止作弊!

1. Find an orthogonal diagonalization of the matrix

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

that is, find an orthogonal matrix C such that $C^{-1}AC$ is a diagonal matrix D.

Answer:
$$C = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \end{bmatrix}$$

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Solution :

6-3 example 4.

2. Prove or disprove that if A and B are orthogonal $n \times n$ matrices, then AB is orthogonal.

Solution :

6-3 example 19(f).