姓名: SOLUTION　　　　　　　　　　　　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ____________　　　　　**Quiz 8**　　　　　考試日期: 2025/04/23

不可使用手機、計算器，禁止作弊!

1. Find the projection matrix for the plane $W: \; x - 3y - 2z = 0$ and then find the projection of $\vec{b} = [2, 1, 3]$ on the plane.

Answer: the projection matrix = $P = \frac{1}{14}\begin{bmatrix} 13 & 3 & 2 \\ 3 & 5 & -6 \\ 2 & -6 & 10 \end{bmatrix}$, $\vec{b}_W = \frac{1}{14}\begin{bmatrix} 35 & -7 & 28 \end{bmatrix}$

Solution：

(Method from 6.4 example 3)

Pick $\vec{a}_1 = [3, 1, 0]^T, \vec{a}_2 = [2, 0, 1]^T$ such that $W = sp(\vec{a}_1, \vec{a}_2)$.

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, (A^TA)^{-1} = \begin{bmatrix} 10 & 6 \\ 6 & 5 \end{bmatrix}^{-1} = \frac{1}{14}\begin{bmatrix} 5 & -6 \\ -6 & 10 \end{bmatrix}$$

The projection matrix P is

$$P = A(A^TA)^{-1}A^T = \frac{1}{14}\begin{bmatrix} 3 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & -6 \\ -6 & 10 \end{bmatrix}\begin{bmatrix} 3 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 13 & 3 & 2 \\ 3 & 5 & -6 \\ 2 & -6 & 10 \end{bmatrix}$$

$$\vec{b}_W^T = P\vec{b}^T = \frac{1}{14}\begin{bmatrix} 13 & 3 & 2 \\ 3 & 5 & -6 \\ 2 & -6 & 10 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \frac{1}{14}\begin{bmatrix} 35 \\ -7 \\ 28 \end{bmatrix}$$

2. Prove one of the following statements: (只需要証一個，記得圈出你要證哪個)

   (a) Show that every real symmetric matrix whose only eigenvalues are 0 and 1 is a projection matrix.

   (b) Show that a projection matrix for a subspace of $\mathbb{R}^n$ has only 0 and 1 as its eigenvalues.

Solution：

(a) 6-4 # 20.

(b) 6-4 # 19(a).