Recall

A : $m \times n$ real matrix $= [a_{ij}]$

1. $A^T = [a_{ji}]$
2. $\vec{u}, \vec{v} \in \mathbb{R}^n$, $\vec{u} \circ \vec{v} = \vec{u}^T \vec{v}$
3. A : orthogonal if $A^T A = I$
4. A : symmetric if $A^T = A$

nxn matrix

$\boxed{6\text{-}3}$

Thm 6.8

Every real symmetric matrix A is diagonalizable by a real orthogonal matrix C.

$$\underbrace{C^{-1}}_{C^T} A C = D \qquad A^T = (CDC^{-1})^T = CDC^{-1} = A$$

A : real symmetric iff A : orthogonal diagonalizable

Def

A : $m \times n$ complex matrix $= [a_{ij}]$

1. $\overline{A} = [\overline{a_{ij}}]$
2. $A^* \equiv (\overline{A})^T = [\overline{a_{ij}}]^T = [\overline{a_{ji}}]$
3. $\vec{u}, \vec{v} \in \mathbb{C}^n$, $\langle \vec{u}, \vec{v} \rangle = \vec{u} \circ \vec{v} = \vec{u}^* \vec{v}$
4. U : unitary if $U^* U = I$
5. A : normal if $A^* A = A A^*$
6. H : Hermitian if $H^* = H$

nxn matrix

$\boxed{\begin{array}{l} \star\ A = [\vec{v}_1\ \vec{v}_2 \cdots \vec{v}_n] : \text{unitary} \\ \because A^* A = I \\ \therefore \vec{v}_i \circ \vec{v}_j = \vec{v}_i^* \vec{v}_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \end{array}}$

$\boxed{9\text{-}3}$

Thm 9.5

Every hermitian matrix H is diagonalizable by a unitary matrix U.

$$\underbrace{U^{-1}}_{U^*} H U = D \qquad H^* = (UDU^{-1})^* = H$$

A : normal iff A : unitary diagonalizable