姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 10**　　考試日期: 2025/11/26

不可使用手機、計算器，禁止作弊!

1. Determine whether the given set S of vectors is dependent or independent. Then reduce the given set to be a basis for $sp(S)$.

$$S = \{1, \sin(x), \sin(2x), \sin(3x)\} \text{ is a subset in a vector space } P.$$

Answer: Is S independent: ($\boxed{Yes}$ / No) .

A basis for $sp(S)$ is $\{1, \sin(x), \sin(2x), \sin(3x)\}$

Solution：

Similar with 3-2 example 3 and 3-2, problem 16.

Suppose $r_1 \times 1 + r_2 \times \sin(x) + r_3 \times \sin(2x) + r_4 \times \sin(3x) = 0$

Solve the equation with $x = 0,\ \pi/6,\ \pi/3,\ \pi/2$, we get $r_1 = r_2 = r_3 = r_4 = 0$.

2. Let B is an ordered tupple $(\sin(x), \sin(3x), \sin(2x), 1)$ and $V = sp(1, \sin(x), \sin(2x), \sin(3x))$. Find a vector in V whose coordinate vector relative to B is $[3, -1, 5, 9]$.

Answer: $3\sin(x) - \sin(3x) + 5\sin(2x) + 9$.

Solution：

$\vec{v}_B = [3, -1, 5, 9]$

Then $\vec{v} = 3 \times \sin(x) + (-1) \times \sin(3x) + 5 \times \sin(2x) + 9 \times 1$

Circle each of the following True or False. Please give a counterexample （反例）for the false statement and give an explain （解釋）for the true statement.

3. True **False** There is a unique coordinate vector associated with each vector $\vec{v} \in V$ relative to a basis for V.

Solution：

3-3, 22(c)

4. True **False** If $\{\vec{v}_1, \vec{v}_2, ..., \vec{v}_n\}$ generates V, then this set of vectors is independent.

Solution：

3-2, 26(e)