姓名: SOLUTION 葉均承 應數一線性代數

學號: ________

Quiz 14

考試日期: 2025/12/24

不可使用手機、計算器，禁止作弊!

1. Using Cramer's rule to find the component x_2 of the solution for the given linear system.

$$\begin{cases} 6x_1 + x_2 - x_3 = 4 \\ x_1 - x_2 + 5x_4 = -2 \\ -x_1 + 3x_2 + x_3 = 2 \\ x_1 + x_2 - x_3 + 2x_4 = 0 \end{cases}$$

Answer: $x_2 =$ 41/59

Solution：

Let $A = \begin{bmatrix} 6 & 1 & -1 & 0 \\ 1 & -1 & 0 & 5 \\ -1 & 3 & 1 & 0 \\ 1 & 1 & -1 & 2 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 4 \\ -2 \\ 2 \\ 0 \end{bmatrix}$. Then $B_2 = \begin{bmatrix} 6 & 4 & -1 & 0 \\ 1 & -2 & 0 & 5 \\ -1 & 2 & 1 & 0 \\ 1 & 0 & -1 & 2 \end{bmatrix}$

Thus, $x_2 = \dfrac{\det(B_2)}{\det(A)} = \dfrac{-82}{-118} = \dfrac{41}{59}$.

2. Let $T : P_2 \to P_3$ be defined by $T(p(x)) = (x-1)p(x+2)$, the ordered basis for P_2 is $B = (x^2 - x, x^2 + x, 1)$ and the ordered basis for P_3 is $B' = (x^3, x^2, x, 1)$. Fine the standard matrix representation A of T relative to the ordered bases B and B'.

Answer: (a) A $\underline{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \\ -4 & -2 & -1 \end{bmatrix}}$, (b) Find the $ker(T) =$ $\underline{\{0\}}$

(c) Given $p(x)$ so that $p(x)_B = [1, 2, 5]$, find $p(x) =$ $\underline{3x^2 + x + 5}$, $T(p(x)) =$ $\underline{3x^3 + 10x^2 + 6x - 19}$

Solution：

$$
\begin{aligned}
&T(x^2 - x) = (x-1)[(x+2)^2 - (x+2)] = x^3 + 2x^2 - x - 2, \\
&T(x^2 - x) = (x-1)[(x+2)^2 + (x+2)] = x^3 + 4x^2 + x - 6, \\
&T(1) = x - 1
\end{aligned}
$$

$$
A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 1 \\ -2 & -6 & -1 \end{bmatrix}, \; rref(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}
$$

By the $rref(A)$, we find the $ker(T)_B = \{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \}$, i.e. $ker(T) = \{0x^2 + 0x + 0 = 0\}$

Let $p(x) = 1(x^2 - x) + 2(x^2 + x) + 5(1) = 3x^2 + x + 5$, $p(x)_B = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$.

$$
T(p)_{B'} = Ap(x)_B = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 4 & 0 \\ -1 & 1 & 1 \\ -2 & -6 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 6 \\ -19 \end{bmatrix}
$$

$T(p) = 3x^3 + 10x^2 + 6x - 19$