姓名: SOLUTION 葉均承 應數一線性代數

學號: ____________ **Quiz 4** 考試日期: 2025/10/08

不可使用手機、計算器，禁止作弊!

1. Let A be a 4×4 matrix. Find a matrix C such that the result of applying the given sequence of elementary row operations to A can also be found by computing the product CA. *Note:* It's fine to leave C unexpanded(不展開).

   The sequence of elementary row operations are:

   (1) Add 6 times row 2 to row 1;

   Solution：

$$\begin{bmatrix} 1 & 6 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (2) interchange row 1 and row 4;

   Solution：

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

   (3) add -3 times row 1 to row 3;

   Solution：

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (4) multiple row 4 by -2.

   Solution：

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

Answer: $C =$ $$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

2. An $n \times n$ matrix A is nilpotent if $A^r = O$ (the $n \times n$ zero matrix) for some positive integer r.
   (a) Given an example of a nonzero nilpotent 2×2 matrix.
   (b) Show that if A is an invertible $n \times n$ matrix, then A is not nilpotent.

Solution：

1-5 problem 29.