姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 9**　　考試日期: 2025/11/19

不可使用手機、計算器，禁止作弊!

1. Consider the set $\mathbb{R}^2$, with the addition defined by $[x,y] \oplus [a,b] = [x+a+2, y+b]$, and with scalar multiplication defined by $r \otimes [x,y] = [rx+r-2, ry]$.

   a. Is this set a vector space? (Yes / *No*)
   Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. If the set is a vector space, then find the zero vector and the additive inverse (加法反元素) in this vector space. *Hint:* The zero vector may NOT be the vector $[0,0]$.
   Answer: the zero vector is **X** , for any vectors [x,y], the -[x,y] is **X**

Solution：

$\vec{0} = 0 \otimes [x,y] = [-2,0]$, and $-[x,y] = (-1) \otimes [x,y] = [-x-3, -y]$

$$[x,y] \oplus (-[x,y]) = [x,y] \oplus [-x-3,-y] = [x+(-x-3)+2, y+(-y)] = [-1,0] \neq [-2,0]$$

It is NOT a vector space.

2. Let V be a vector space. Prove that, if $\vec{v}$ is in V and if r is a scalar and if $r\vec{v} = \vec{0}$, then either $r = 0$ or $\vec{v} = \vec{0}$.

Solution：

3-1, problem 23.