Quiz 10

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Express $\frac{3+4i}{2+5i}$ in the form $a+bi$, for a and b real numbers.

   Answer: $\frac{26}{29} - \frac{7}{29}i$

   Solution：

$$\frac{3+4i}{2+5i} = \frac{(3+4i)(2-5i)}{(2+5i)(2-5i)} = \frac{6-15i+8i-20i^2}{2^2+5^2} = \frac{26-7i}{29} = \frac{26}{29} - \frac{7}{29}i$$

2. Express $(\sqrt{3}-i)^{20}$ in the form $a+bi$, for a and b real numbers.

   Answer: $-2^{19} + 2^{19}\sqrt{3}i$

   Solution：

   Polar Form: $\sqrt{3} - i = 2\left(\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6})\right)$

$$(\sqrt{3}-i)^{20} = 2^{20}\left(\cos\left(-\frac{20\pi}{6}\right) + i\sin\left(-\frac{20\pi}{6}\right)\right) = 2^{20}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$$= 2^{20}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -2^{19} + 2^{19}\sqrt{3}i$$

3. Find all the possible complex z such that $z + 2\bar{z} = 3z$.

Answer: $z \in \mathbb{R}$ **(all real numbers)**

Solution：

Ch 9-1, problem 17 (j).

Let $z = a + bi$, then $\bar{z} = a - bi$:

$$(a + bi) + 2(a - bi) = 3(a + bi)$$

$$3a - bi = 3a + 3bi \implies 4bi = 0 \implies b = 0$$

Since $b = 0$ and a can be any real number, thus z can be any real number.