姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 2**　　考試日期: 2026/03/11

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Given a matrix A and use it to answer the following question.

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 0 & 4 & 0 \\ -1 & 1 & 2 \end{bmatrix}$$

(a) find the eigenvalues and a corresponding eigenvectors of A: $(3, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}), (4, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix})$.

(b) Find a matrix C and a diagonal matrix D such that $AC = CD$.

Answer: C= $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix}$, and D= $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

(c) Is A diagonalizable? (Yes / $\boxed{NO}$) Why? **見下方理由，沒寫理由不給分**

Solution：

Since eigenvalues of A is $3, 3, 4$ and their corresponding eigenvector are $(3, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}), (4, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix})$.

(C) 利用 Theorem 5.4 能知道 A 不是 diagonalizable，因為 eigenvalue 3 的 a.m.=2，但是 g.m. 是 1 。

2. Prove or disprove the following statements:

   If A and B are similar square matrices and A is diagonalizable, then B is also diagonalizable.

   Solution：

   5-2 problem 13(h).

   It is True!