

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Let the sequence a_0, a_1, a_2, \dots be given by $a_0 = 2, a_1 = 5$ and $a_k = 5a_{k-1} - 6a_{k-2}$ for $k \geq 2$. Find a_n .

Answer: $a_n = \underline{2^n + 3^n}$

2. Solve the given system.

$$\begin{cases} x_1' = 3x_1 - x_2 + x_3, \\ x_2' = 2x_2 , \\ x_3' = x_1 - x_2 + 3x_3 \end{cases},$$

Answer:
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} k_1 e^{4t} \\ k_2 e^{2t} \\ k_3 e^{2t} \end{bmatrix} = \begin{bmatrix} k_1 e^{4t} - k_2 e^{2t} + k_3 e^{2t} \\ k_3 e^{2t} \\ k_1 e^{4t} + k_2 e^{2t} + k_3 e^{2t} \end{bmatrix},$$

Solution :

Note that the solution is not unique!