1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Let $\vec{v_1} = [1,1,0], \vec{v_2} = [1,2,0], \vec{v_3} = [0,1,2]$. Using the Gram-Schmidt process to find an orthonormal basis for $\mathbb{R}^3$ and find a QR-factorization of A, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

Answer: The $Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$, The $R = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 2 \end{bmatrix}$

the requested orthonormal basis is $\frac{1}{\sqrt{2}}[1,1,0], \frac{1}{\sqrt{2}}[-1,1,0], [0,0,1]$

Solution：

Let $\vec{a_1} = [1,1,0]$, $\vec{a_2} = [1,2,0]$, $\vec{a_3} = [0,1,2]$,

$$\vec{v_1} = [1,1,0], \qquad \vec{q_1} = \frac{\vec{v_1}}{\|\vec{v_1}\|} = \frac{1}{\sqrt{2}}[1,1,0],$$

$$\vec{v_2} = \vec{a_2} - \frac{\vec{v_1}\cdot\vec{a_2}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1} = \frac{1}{2}[-1,1,0], \qquad \vec{q_2} = \frac{\vec{v_2}}{\|\vec{v_2}\|} = \frac{1}{\sqrt{2}}[-1,1,0]$$

$$\vec{v_3} = \vec{a_3} - \frac{\vec{v_1}\cdot\vec{a_3}}{\vec{v_1}\cdot\vec{v_1}}\vec{v_1} - \frac{\vec{v_2}\cdot\vec{a_3}}{\vec{v_2}\cdot\vec{v_2}}\vec{v_2} = [0,0,2], \qquad \vec{q_3} = \frac{\vec{v_3}}{\|\vec{v_3}\|} = [0,0,1],$$

For the matrix $A = [\vec{a}_1^T, \vec{a}_2^T, \vec{a}_3^T] = QR$ decomposition:

$$Q = [\vec{q}_1^T, \vec{q}_2^T, \vec{q}_3^T] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$R = \begin{bmatrix} \vec{q}_1\cdot\vec{a}_1 & \vec{q}_1\cdot\vec{a}_2 & \vec{q}_1\cdot\vec{a}_3 \\ 0 & \vec{q}_2\cdot\vec{a}_2 & \vec{q}_2\cdot\vec{a}_3 \\ 0 & 0 & \vec{q}_3\cdot\vec{a}_3 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 2 \end{bmatrix}.$$