姓名: SOLUTION　　　　　　　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________

Quiz 8

考試日期: 2026/04/29

不可使用手機、計算器，禁止作弊!

1. Find the change-of-coordinates matrix from B to B' and from B' to B, indicate which is which, and use it to find the coordinate vector $\vec{v}_{B'}$ with

$$B = \{\mathbf{b}_1, \mathbf{b}_2\} = \left\{ \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \end{bmatrix} \right\}, \quad B' = \{\mathbf{b'}_1, \mathbf{b'}_2\} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}, \quad \vec{v}_B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Answer: $C_{BB'} = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}$, $C_{B'B} = -\frac{1}{2}\begin{bmatrix} 2 & -1 \\ -2 & 0 \end{bmatrix}$, $\vec{v}_{B'} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$.

Solution：

$$M_B = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}, \quad M_{B'} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

Using row reduction:

$$[M_{B'} \mid M_B] \sim [I \mid C_{B,B'}]$$

or by formula:

$$C_{B,B'} = M_{B'}^{-1} M_B = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix}$$

$$C_{B',B} = C_{B,B'}^{-1} = -\frac{1}{2} \begin{bmatrix} 2 & -1 \\ -2 & 0 \end{bmatrix}$$

$$\vec{v} = M_B \vec{v}_B = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\vec{v}_{B'} = C_{B,B'} \vec{v}_B = \begin{bmatrix} 0 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

2. Prove that if $C_{B,B'}$ is an orthogonal matrix and B is an orthonormal bases, then B' are is is an orthonormal bases.

   Solution：

   7-1 #23(e)