姓名: SOLUTION　　葉均承　　應數一線性代數

學號: _____　　**Quiz 4**　　考試日期: 2026/10/07

不可使用手機、計算器，禁止作弊!

1. Let $A = \begin{bmatrix} 1 & 6 & 0 & -1 \\ 0 & 1 & 0 & 8 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. If possible, find a matrix B such that $AB = A^2 + 5A$.

Answer: $B = \begin{bmatrix} 6 & 6 & 0 & -1 \\ 0 & 6 & 0 & 8 \\ 2 & 0 & 6 & 0 \\ 0 & 1 & 0 & 6 \end{bmatrix}$

Solution：

Pick $B = (A + 5I)$, then

$$AB = A(A + 5I) = A^2 + 5A.$$

2. Let $A^{-1} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix}$. If possible, find a matrix B such that $AB = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix}$.

Answer: $B = \begin{bmatrix} 5 & 5 \\ 4 & 4 \\ 12 & 11 \end{bmatrix}$

Solution：

$$B = (A^{-1}A)B = A^{-1}(AB) = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 4 & 4 \\ 12 & 11 \end{bmatrix}$$

3. Let A be a matrix such that A^2 is invertible. Prove that A^3 is invertible.

Solution：

1-5 problem 23(e).